HULL SPEED

AN INSIGHT INTO THE PRINCIPAL BARRIER
TO HIGH SPEED UNDER SAIL

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PREFACE

Sailboat design is an art and, as for all art, success demands not only the development of natural talent but also an intimate knowledge of the subject. For sailboat design, the accumulation of that knowledge has been a lengthy, evolutionary process, beginning in prehistory and achieving a significant level of advancement by the time of man's earliest records. The proas and double canoes of Oceania, the junks of Asia and the Viking ships of Europe typify the extraordinary refinement and diversification of evolutionary design evident throughout the history of the maritime world. Although aided by increasingly sophisticated technology, modern sailboat design is essentially an extension of that evolutionary process.

Today, the theory of all boat design is regarded as an engineering science, whereby the behaviour of a vessel is explained by the application of mathematics. However, some factors that determine the performance of a boat, especially under sail, are still seemingly impossible to resolve other than by empirical methods. Consequently, the theory of certain aspects of sailboat design remains limited to broad principles, the interpretations of which are dependent on the adeptness of the designer.

Shaping a hull to reduce resistance is the most fundamental, most studied but generally the least understood feature of design for all types of vessels.

Surface waves caused by the forward motion of a hull represent a major component of a vessel's total resistance. But, despite being the most visible form of resistance and the principal barrier to high speed under sail, the causes and effects of wave-making are commonly misunderstood. Although detailed analyses of wave-making resistance are to be found in advanced texts on hydrodynamics and undoubtedly have their elaborate mathematical solutions in use in today's proliferation of hull design software, very little of that technical knowledge seems to have filtered through to the general world of boating by way of sensible, logical explanation.

***Hull Speed** is not intended as an expert analysis of the resistance caused by the formation of waves but **is** the product of one layman's endeavour to gain an insight into what has always been a most challenging aspect of sailboat design, shaping a hull to reduce wave-making resistance.*

*Presented in two parts, the first, **From Theory to Evolution,** is introductory, an historical prelude to **A Component Waveform Theory**, tendered as a fresh, unique and relatively uncomplicated interpretation of wave-making to demonstrate the effect that wave-making has on the performance of a vessel, how wave-making creates a barrier to high speed and, finally, how the resistance due to wave-making might be minimised.*

At the outset, I readily admit to being unqualified to attempt the analysis of a topic of such complexity, my interest in sailboat design simply stems from an inexplicable lifelong obsession with small boats and sailing. Inevitably, the use of a novel 'component waveform theory' to simplify the technicalities of wave-making is likely to be dismissed as being a very naive approach to an exceptionally complex mathematical problem. It is, but the concept, no matter how elementary or flawed, does help to clarify for laymen an otherwise extremely vague area of hull design.

*Essentially, **Hull Speed** is the outcome of a project that began in the early 1970s as an attempt, by me, to design an NS14, an Australian small sailing dinghy capable of efficient planing. Being a novice to sailboat design, my original intention was merely to gain an understanding of the hull design process but almost immediately, because of an apparent lack of useful theoretical design information available, the exercise became, instead, a personal quest to derive a mathematical method for determining the optimum 'curve of areas' for the immersed sections of a hull travelling at any speed, including speeds above 'hull speed'.*

To my knowledge, which remains very wanting in these matters, no one had previously achieved such an objective. Adding to my uncertainty at the time, the science of planing hulls was depicted in the boating publications of the day as almost a distinct discipline, somewhat detached from the traditional approach to the design of displacement hulls. Although that point of view had always seemed strange to me, young and practically ignorant of the history and technicalities of design, there appeared to be unanimous agreement amongst experienced designers of a virtual collapse of traditional design theories once planing began, an opinion that still seems to hold sway today, some four decades later.

Nevertheless, having already witnessed first-hand the smooth transition of NS14 sailing dinghies from displacement to planing speeds and convinced that the supposed discontinuity in design philosophy that occurred at 'hull speed' was implausible and seemed to lack a solid scientific basis, I persisted, developing my own lines of thought. During the next couple of years, while struggling to untangle the complexity of wave-making, I inadvertently developed the Component Waveform Theory, a logical approach to minimising the wave-making resistance of a hull at any practical speed, an approach that is straightforward and understandable, in the spirit of the early 'amateur' theorists such as John Scott Russell and Colin Archer, whose theories on how to minimise wave-making resistance are included in the prelude.

Subsequent investigation has led to my realisation that all of the know-how necessary to develop a component waveform theory was already common knowledge to designers when boats capable of planing were in the early stage of development, more than one hundred years ago, in the first decade of the 20th century. However, since independently developing the component waveform concept intuitively almost four decades ago, I have yet to encounter a more persuasive account of 'hull speed' or of wave-making resistance generally in any boating publication, old or new. My ongoing exploration of the past and present 'state-of-the-art' of boat design has, if anything, increased my confidence in the Component Waveform Theory which, for laymen at least, has the potential, I believe, to rationalize the conventional approach to sailboat design, hence the compulsion to subject the theory to criticism in the public arena.

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September, 2014

INDEX

FROM THEORY TO EVOLUTION

The Age of Sail
A New Beginning
The Trochoidal Wave Theory
A Waveline Theory
The Wake
A Waveform Theory
A Return to Evolution

A COMPONENT WAVEFORM THEORY

The Component Wave
Hull Speed
Below Hull Speed
Semi-planing and Planing
The Theory Applied
Fact or Fiction?

APPENDIX

A Design for a Dinghy

FROM THEORY TO EVOLUTION

THE AGE OF SAIL

A sailboat is unique. Travelling at the interface of two media, a sailboat is supported by the water, continually pitching, rolling, yawing, surging, heaving and heeling on that fluid's unpredictable surface while gaining propulsion from the air above, equally unpredictable. Attempts to study sailboat design, even at an elementary level, reveal a complex and indefinite topic. Design possibilities are infinite, exemplified in a broad sense by the multiplicity of traditional and commercial sailing craft that have evolved worldwide over countless generations of designers, builders and sailors. The modern sailboat, having its infancy in the decline of commercial sail during the nineteenth century, is a product of that evolutionary process.

The origins of sail precede recorded history, possibly by many thousands of years. Archaeological evidence suggests that as early as the 4th millennium BC, commercial sailing vessels, as distinct from more ancient traditional craft, were in use on the Nile River in Egypt. Similar but unrecorded developments were undoubtedly occurring independently throughout Asia. From that period in the late Stone Age, until the Industrial Revolution more than five thousand years later, the sailing ship evolved at the forefront of man's technological achievements. Influenced by the demands of trade, exploration and warfare, development was generally cautious and deliberate, interspersed with the occasional revolutionary innovation and constantly reflecting the ingenuity of the societies involved.

Constructed of bundles of reeds, lashed together and bent to form the shape of the hull, the first Egyptian ships were rowed with oars and steered with oars at the stern, a square sail being used for running downwind. These vessels were a development of smaller traditional craft that had been in use by Neolithic man on the Nile for thousands of years. Such had been the impact of reed boats on society that apart from weapons, they are reputed to be often the only manufactured product depicted in early Egyptian art. Although seemingly primitive, reed construction enabled vessels to be quickly and easily built from readily available materials and by using the simplest of tools. Testimony to the adequacy of reed construction and to the refinement of design possible within a stone-age culture is still to be found in several parts of the world and as remote from Egypt as Lake Titicaca, the world's highest navigable waterway, shared by Bolivia and Peru in South America. Located on a high plateau in the Andes Mountains, the area surrounding Lake Titicaca is bleak and treeless. Totora reeds growing along the shoreline have long provided the indigenous population with its only source of local boatbuilding material and, even today, using techniques strikingly similar to those of the ancient Egyptians, the reeds are meticulously fashioned into small traditional boats that are not only superbly functional works of art but have the advantage of being inherently buoyant. Usually poled through the shallows, the boats can be sailed downwind like their Egyptian counterparts, using a sail woven from the reeds.

Demand for larger commercial vessels in ancient Egypt paralleled the emergence of a more complex social structure. Before the 5th millennium BC, settled agriculture had begun on the fertile banks of the Nile River and by the first half of the 4th millennium BC villages were becoming permanent as shifting cultivation and the herding of sheep and cattle were developed. Craftsmen were able then to devote more time to enhancing skills in weaving, pottery and tool making. Mud brick gradually replaced the wattle and daub construction of earlier buildings, population increased and trade expanded significantly. Towards the end of the 4th millennium BC

Egypt had become a unified kingdom, importing raw materials for building and tool making from neighbouring regions. During this period of increasing economic and cultural activity innovations occurred that would have the most profound effects on human development. Papyrus, the reed commonly used for boat construction, including the sails and rigging, and for manufacturing such diverse items as baskets, mats and sandals, was to take on a most important role, as a writing material. The Egyptians had also devised a simplified script that could be written on the paper with ink, a distinct improvement over the previous method of carving on clay tablets. However, it was the discovery that metallic copper could be produced from ore, leading to the technology of making bronze, an alloy of copper and tin, that was to have a more immediate impact on society, influencing traditional crafts, especially carpentry.

Throughout the 3rd millennium BC, it is thought that bronze technology began to spread, first to the eastern Mediterranean and eventually westward towards Europe, instigating a period of great social change. A trading network in raw materials, designs and techniques in bronze work spread rapidly, dispersing products extensively, eventually creating an almost international style in tools, weapons, ornaments and household items. However, under the reign of the Pharaohs Egyptian society remained largely rural, with most of the population living in small villages. Despite the extraordinary engineering feats of the construction of the pyramids and networks of irrigation canals, projects that employed thousands of men for extended periods, cities became centres of government but not of manufacturing or commerce. At sea, although Egyptian reed boats had been venturing offshore into the Mediterranean since before the millennium, Egypt was not destined to become a formal maritime power.

To the north-west of Egypt, fortuitously located to take advantage of the expanding trade between the developing civilisations of south-west Asia and the still nomadic pastoralists of eastern Europe, lay the Mediterranean island of Crete, upon which Minoan society was already surfacing. Regarded by historians as one of the great early civilisations, the Minoans were prominent from about 3000BC to 1350BC, developing concurrently with their Egyptian neighbours. The island population lived in villages, carrying on mixed farming and small industrial enterprises. Lime was extracted for plastering their timber and stone buildings, paints and dyes were made from plants and earth colours, water supplies were piped. Skilled craftsmen produced highly polished pottery and beautiful jewellery made of gold and semiprecious stones. Trade of pottery and textiles with the mainland provided essentials such as obsidian, from which stone tools were fashioned. For the Minoans sea-going vessels were essential, not only for fishing and transport, but as the sole means of external communication.

Ships constructed of papyrus reeds were the mainstay of the early international trade and Egyptian artwork from that period suggests vessels of considerable size, some having crews of fifty or more oarsmen, while others were so large that a second deck was constructed above the first. Timber, however, has always been the global boatbuilding material. Primitive rafts have been made by lashing logs together, small craft have been constructed by stretching animal hides over flimsy timber frameworks, canoes have been fashioned from bark or shaped with stone axes from logs, hollowed by burning and then scraped with rocks or shells. A by-product of the new bronze technology was the stimulus given to timber boatbuilding, through the availability of good bronze tools. Not surprisingly, with the introduction of metal tools to the island of Crete, the Minoans quickly sought to improve their boatbuilding techniques. By 2000BC, in contrast with the mainland Syrians and Egyptians who used the new technology to improve overland transport by developing the spoked wheel, the Minoans were building wooden planked hulls, revolutionising timber boatbuilding techniques and pioneering the way for larger and more seaworthy ship construction.

At first, the hull lines of the timber merchantmen simply replicated those of reed ships, which had already evolved for more than a millennium into an efficient and seaworthy shape capable of transporting large quantities of cargo along the Mediterranean coasts. For centuries, both types of vessels co-existed, the reed ship having the advantage of inherent buoyancy while the wooden vessel, although more difficult to construct, offered longevity and the possibility of further development. The design of sea-going ships for specific purposes was by now becoming an important undertaking and, in time, the Minoans devised the narrower and faster fighting ship fitted with a protruding ram forward, using a single mast to carry a square sail for cruising and depending on oars for speed and manoeuvrability in battle. Inevitably, as trade with Egypt expanded, the superiority of Minoan vessels allowed Crete to emerge as a naval power, ultimately controlling the trade routes of the eastern Mediterranean Sea.

From about 2000BC Minoan society flourished, revealed by the growth of highly adorned palaces as the centres of power. Throughout the island kingdom the palaces were the great administrative centres, responsible for the management of trade and the production, storage and distribution of food. Manufacture of high quality metalwork, jewellery and pottery continued and during this period new industrial techniques were added. The potter's wheel was introduced and pottery decoration developed to equal the most artistic of any place or time. The scene was one of unprecedented economic growth, unexpectedly interrupted around 1600 BC by some form of total destruction, thought now to be a violent earthquake. Despite the catastrophe, the Minoans rebuilt on a more luxurious scale, developing and perfecting earlier styles, and achieving an even greater period of prosperity. Suddenly, around 1500BC, Crete's buildings were destroyed again, possibly by a distant volcanic eruption and subsequent tsunamis which severely affected the island's shipping and the economy. Reconstructed once more, a weakened and defenceless Crete was conquered soon after by the Mycenaean, a neighbouring race of wide-ranging traders and looters from the Greek mainland. Ultimately, the Minoan palaces were destroyed by fire, possibly marking a final uprising of Cretans against Mycenaean domination. The brilliance and splendour of the Mediterranean's foremost maritime nation had vanished forever.

Natural disasters and political upheavals of exceptional magnitude continued to influence the distribution of power in the Mediterranean, destroying all of the formerly great civilisations within a period of two centuries. The Mycenaean, who had absorbed the Minoan culture and dominated the sea lanes after the destruction of Crete, were overwhelmed by several invasions from the north. In Egypt, internal economic unrest and corruption had seriously weakened the government and the Mediterranean coastline came under attack by various groups of aggressive seafaring invaders known collectively as Sea-People. The end of the true age of the Pharaohs was imminent and the Egyptians were never to regain their importance. To the east a similar fate awaited the Hittite empire of Anatolia, the Asiatic portion of modern Turkey. Whole populations fled in search of new lands, taking with them the crafts and skills each had developed over centuries. Former secret discoverers of the process of crafting quality iron, the superiority of Hittite weapons and tools was immediately recognised as the refugees from Anatolia invaded neighbouring Syria and Palestine. Agriculture, manufacturing and warfare were about to be revolutionised once more. The knowledge of iron metallurgy spread rapidly and, unlike the more expensive bronze, made possible the large scale production of metal tools and weapons, allowing more permanent settlement than before and prompting a mass movement of populations that would change the face of Asia and Europe.

Throughout the eastern Mediterranean the transition from the Bronze Age to the Iron Age had occurred during a period of unprecedented political turmoil and nowhere is that more evident than in the vicinity of Palestine, where many events were recorded in biblical literature. By the

13th century BC Egypt's dominance over its neighbours was weakening and during that century the Hebrews, under the leadership of Moses, made their escape from slavery in Egypt and migrated eastward in search of the Promised Land. Archaeological and historical evidence suggests the process of conquering the land was lengthy and not completed until early in the 10th century BC when Jerusalem was captured and a united kingdom established over Israel. The former occupants of Palestine had been predominantly Canaanite and their territory extended beyond the Israelite influence, along the narrow coastline of modern Lebanon and into Syria. Effectively, from the time of the Israelite occupation of Palestine, the remaining Canaanite territory has been referred to by historians as Phoenicia, a name bestowed on it by later generations of Greeks.

The location of Phoenicia at the junction of the land and sea routes between Asia and the Mediterranean coasts had led to the rise of a mercantile society. Commercial links by sea with Egypt can be traced from the 3rd millennium BC and throughout their history the Phoenicians were generally regarded by their contemporaries as skilled traders, using commerce as their principal motivation and source of influence. However, the Phoenician contribution to the advancement of civilisation extends far beyond their role as mere buyers and sellers. An alphabet of twenty-two letters was in use by the Phoenicians as early as the 15th century BC and being subsequently adopted by the Greeks has become the ancestor of the modern Roman alphabet. In addition to the alphabet, the introduction of a standard system of weights and measures helped the Phoenicians to establish their commercial supremacy. To maintain that supremacy the Phoenicians had also developed skills in shipbuilding, navigation and seafaring.

Large wooden ships to be used for trading purposes were probably first built on the shores of Phoenicia. Impetus for the growth of the earlier Minoan civilisation during the 3rd millennium BC had initially come through trade with the eastern Mediterranean and unlike the Minoans and the Egyptians, for whom timber was scarce, the Phoenicians had access to abundant timber reserves, particularly cedar and pine. Interestingly, one of the world's oldest known shipwrecks is believed to be a Phoenician or Syrian trading vessel, which had sunk in about 1200BC off the coast of Turkey, carrying copper ingots from Cyprus and hundreds of bronze tools. The spread of metal technology during the 3rd millennium BC and the subsequent development of wooden hulls had led to an increasingly complex trade network and as deposits of copper and tin were discovered, traders, particularly the Phoenicians, travelled long distances to purchase the rare and precious metals. Uncontested at sea, following the abrupt demise of the Minoans and their successors, the Mycenaean, the Phoenicians were destined to become the next maritime power.

Designed for commercial viability, Phoenician trading vessels emphasised cargo capacity and ease of handling with a small crew at the expense of speed. Varying in length to about 30 metres, these heavily-built, wooden-planked ships were beamy with a pronounced sheer and carried a single square sail on a single pole mast. Oars were used for secondary propulsion and steering oars were mounted aft, both port and starboard. In contrast, Phoenician warships exhibited a Cretan influence, being narrow and fast, with the now familiar protruding ram forward. Oars were mounted in a two-tier arrangement, increasing the rowing power and consequently, the speed of the vessel. A single square sail was used for cruising.

From about 1100BC, for three centuries, the Phoenicians dominated the trade routes of the eastern Mediterranean Sea. Intrepid navigators, they patiently ventured further westward where no one else dared, establishing protected trading ports along the north coast of Africa. Carthage, the most famous of the African ports, became the centre of maritime power in the western Mediterranean and through its trade, probably the richest city in the world. The quest for new commodities, particularly minerals, eventually led to the establishment of Carthaginian colonies in southern Spain and evidence suggests that Carthaginian vessels may have sailed as far north as Britain, southward along the west coast of Africa, westward to the Azores and possibly to Madeira and the Canary Islands. A Phoenician fleet, commissioned by the Egyptian pharaoh and sailing from the Red Sea, successfully attempted a three-year circumnavigation of the African continent at the end of the 7th century BC, evidence of the skills in seafaring and navigation for which the Phoenicians had become famous. The Phoenicians accumulated a knowledge of winds and currents that enabled the establishment of regular trade routes and are credited with developing the use of the star, Polaris, as a navigational tool for the establishment of latitude. Phoenician domination of the Mediterranean Sea was not seriously challenged until the evolution of the Greek galley during the 8th century BC. By the end of the 4th century BC the Phoenician homeland had been absorbed by the Persian Empire, although the Phoenician colony of Carthage continued to dominate trade in the western Mediterranean for another two centuries until ultimately destroyed by the Romans.

Phoenician trade with the Greek world had been substantial from the outset and the mingling of the two cultures inevitably led to a period of development and growth. By the mid 8th century BC the Greeks had begun to move out of the Aegean to settle on the coasts of the Black Sea and the Mediterranean. Athens and Sparta emerged as the principal cities, each being an almost entirely independent entity, ruled by kings and with a developing aristocracy. The poorer classes had no political rights and were often sold into slavery but by the beginning of the 7th century BC, in the first step towards democracy, Sparta had handed ultimate political power to a citizen assembly. A century later slavery was abolished in Athens. By then the Greeks had their own alphabet and the development of coinage led to increased economic activity and a general rise in the standard of living. Philosophy and science flourished in Greece from about this period.

Pythagoras, a Greek philosopher and mathematician from the 6th Century BC, is credited with developing the mathematical theory that *'in a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides'*. His adherents were also the first to suggest that the sun, Earth and the other planets are spheres revolving in the heavens. Four centuries later Eratosthenes, a geographer and astronomer, was able to estimate the circumference of the Earth. By then, the sciences were well advanced and Greek culture was spread over a larger area than ever before. Euclid, a Greek mathematician from the 3rd Century BC, excelled in a variety of topics including astronomy, geometry and optics. His most important works, thirteen books known collectively as *The Elements*, represented an accumulation of all of the Greek mathematical knowledge of the period and dealt with plane geometry, three-dimensional geometry and the theory of numbers. Euclid's works were to become the standard treatise on mathematics throughout Europe until the 19th century. Archimedes, also from the 3rd century BC, is probably the most famous of all the Greek scientists and mathematicians. His mathematical works included the calculation of the area of a circle and other enclosed curves and he wrote works on theoretical mechanics and hydrostatics. Archimedes was also an inventor of many practical mechanical devices, particularly the hydraulic screw, a spiral pump designed to raise water to a higher level, usually for irrigation purposes. However, it is for the scientific principle, that he supposedly discovered while floating in a bath, that Archimedes is mostly known today. That discovery, Archimedes' Principle, which simply states that *'a body immersed in a fluid appears to lose weight, and that loss of weight is equal to the weight of fluid displaced'* has become the cornerstone of modern naval architecture.

Greek merchant vessels were similar to the Phoenicians' but the evolution of warships in the Mediterranean between the 8th and 3rd centuries BC paralleled the ascent of the Greek civilisation. To provide protection from the sea for their developing coastal cities the Greeks devised the galley, a vessel built primarily for fighting and whose lines were probably adopted from the Phoenician warships. The first galleys, known as uniremes, carried a single bank of oars and were mostly undecked. The bireme, a galley with two banks of oars, soon followed and was to become the leading warship of the period. Biremes had an overall length of about 25 metres and a maximum beam of only 3 metres. Further development of the galley resulted in the trireme, carrying three banks of oars. The trireme was to become the primary warship by the 5th century BC and typically had a length of about 40 metres, a beam of 6 metres and a draft of 1 metre. The vessels were crewed by as many as two hundred men, including officers, seamen, oarsmen and heavily armed marines. The oarsmen, the majority of the crew, sat on three levels and slid back and forth on leather cushions to utilise the full strength of their thigh muscles and under oars it is estimated that triremes may have attained speeds of 10 knots. Like its predecessors, the trireme carried a sailing rig consisting of a single mast fitted with a square sail, the entire rig being lowered and stowed when rowing upwind or into battle. Warships such as the trireme could not remain at sea for long periods because of their limited size and large crews, voyages usually consisting of short coastal hops.

The ram remained the primary weapon of the trireme as it was for the Phoenician and the earlier Minoan warships. Usually plated in bronze, the ram was an elongation of the heavy keel that formed the backbone of the vessel. For the trireme extra longitudinal strength was achieved by the construction of a gangway along the centreline of the vessel. In battle the gangway was used by the crew as a storming bridge to board the enemy vessel after it was rammed. As ships increased in size and became heavier, the ram became less effective and boarding assumed greater importance. By the beginning of the 3rd century BC the race for superiority at sea led to the design of larger warships in the eastern Mediterranean, some vessels requiring crews of more than two thousand men. The introduction of heavy missile weapons that could catapult darts and stones meant that for the first time the enemy could be engaged from a distance. Ironically, the arms race at sea was ultimately brought to an end by the might of an expanding Roman Republic.

The Romans were not traditional seafarers. From the outset, instead of warships, highly disciplined and well-equipped armies had been used to extend the borders of the ambitious republic. However, after suffering defeats on the island of Sicily and a series of conflicts with Carthaginian fleets in the 3rd Century BC, the Romans moved to rectify their disadvantage. A fleet of galleys was hastily constructed and to counter the enemy's experience at sea the Romans adapted the tactics of land warfare at which they were extremely successful. The number of marines onboard was increased and the galleys were modified to allow a hinged gangplank with a grappling hook to be fitted in the bows. In battle the opponent was rammed and the gangplank lowered, locking the warships together and allowing the marines to board the enemy vessel and overwhelm the crew. Boarding was to become the primary tactic and proved to be extraordinarily successful, although costly in terms of human life. In one decisive naval battle off the coast of Sicily, the Roman fleet was able to destroy or capture more than forty Carthaginian ships and ten thousand men. By the end of the 3rd Century BC, through sheer force, Rome had become the leading sea power within the Mediterranean, a position it would maintain for more than a millennium.

Most of the old shipwrecks found in the western Mediterranean are from the Roman era. As the Republic expanded into a vast empire and acquired new overseas territories, commercial shipping soon followed. Cargoes varied from agricultural produce to decorative objects such as marble columns, bronze and marble statues and fine furniture. A typical bluff-bowed cargo vessel of the

1st century BC exceeded 30 metres in length, with a beam of about 9 metres. In addition to the usual rig consisting of a single mast fitted with a square sail, a foremast carrying a much smaller sail was sometimes stepped in the bows to assist in steering, which was still controlled primarily by two oars fitted at the stern. To sail to a chosen destination these heavy-displacement, square-rigged vessels were nevertheless restricted to sailing downwind, offering little improvement in that regard to the reed boats of the Egyptians some three thousand years earlier. But, within a few centuries, during the early Christian era, a radical innovation in rig design emerged from the Arab world. More likely the result of intuitive experiment rather than gradual evolution, the triangular lateen rig made it possible for the first time, in the Mediterranean at least, for suitably designed vessels to sail closer to the wind. The lateen rig proved to be particularly successful on small craft where local conditions were favourable and has been characteristic of Arab sailing vessels to the present day. However, on the larger vessels of the Mediterranean fleets the square sails were not discarded entirely.

A square sail set on a single mast was also the rig used by the Norsemen, the 'people from the north' who occupied the north-west coast of Europe. The Romans had regularly sailed as far west as Britain since the 1st century BC and were aware of the northern European boats, which archaeological evidence suggests had been influenced in their design by early Mediterranean warships. By the 8th century AD Norse boats had evolved into the most advanced ships of the known world. The western Roman Empire had deteriorated and a new era, the Viking Age, was emerging. During that period, from the late 8th century to the 11th century, the Norsemen gained an infamous reputation for their devastating raids on the coasts and rivers of Europe, including the Mediterranean, and parts of northern Africa. Today we refer to those sea-borne warriors as Vikings and to their famous ships, as Viking longships.

Norsemen, generally, were not Vikings but farmers, hunters, craftsmen and traders, living mostly in isolated communities scattered along the coastlines of modern-day Norway, Denmark and Sweden. Overland travel between settlements was hampered by the rugged terrain and, as a consequence, coastal vessels capable of handling the open seas of the north were indispensable for transport, trade and defence. Travel by water was a way of life, and the Norsemen became accomplished seamen and masters of the art of timber shipbuilding. In the centuries preceding the Viking Age, the Norsemen had developed two distinct types of sea-going vessels, the knaar and the longship. Built to haul cargo over long distances, the beamy knaars were the more numerous, the longships being used primarily for coastal defence against enemy attack which, at times, included Vikings, who were not always an accepted part of Norse society.

A typical knaar had an overall length of about 15 metres, a beam of 5 metres and a draft of 1 metre, although remains of vessels ranging in lengths from as little as 10 metres to more than 20 metres have been discovered. A relatively short mast, stepped almost amidships, carried a single square sail made from wool or linen, set from a long yard to provide a substantial sail area. Oars fitted in the bow or stern were probably only used for manoeuvring in close quarters since crews numbered as few as five men. The more refined longships were, as the name suggests, generally longer than the traders, but were also lighter, narrower, shallower and designed for speed. Oars fitted to almost the entire length of the hulls were the main form of propulsion in situations where speed and manoeuvrability were essential, particularly in combat. At sea, single square sails enabled the vessels to cover long distances faster than by oar. Longships varied from about 17 metres in length, coupled with a beam of 2.5 metres, a draft of 0.5 metre and crewed by twenty-five men in the most common vessels, to double that size, having an overall length of about 35 metres, a beam of 5 metres, a draft of 1 metre and crewed by more than one hundred men. A fleet of as many as one hundred and twenty longships is known to have been used in one

Viking raid on the coast of Ireland during the 9th century. On raids, the Vikings used their longships as troop carriers rather than warships, although the vessels were sometimes lashed together to form a steady platform for fighting.

Both types of vessels, the knaar and the longship, were double-ended, undecked, clinker-planked, steered with a rudder attached to the side of the ship near the stern and in design, construction and performance surpassed the earlier sailing ships of the Mediterranean, which were cumbersome by comparison. Although more than a millennium has elapsed since their inception, the balanced underwater lines of the Norse vessels compare favourably with those of modern designs and replicas of both the knaar and the longship have been constructed and proven to be seaworthy, fast off the wind and capable of making slow but steady progress to windward. Lateral resistance was provided by a long, deep keel and an intentional steepness of the hulls' lower planking, enabling the vessels to point as high as sixty degrees from the wind direction but with considerable leeway in strong winds. The rudder, of balanced design, was an exceptional innovation that assisted windward performance and in the modern replicas has been found easy to use in all conditions. The legendary seamanship of the Norsemen and the performance of their ships at sea are substantiated by the fact that they were able to establish and maintain contact with communities in the Faroe Islands, the Shetlands, the Orkneys, Iceland, Greenland, Ireland and Great Britain. Countless voyages were also undertaken in the Baltic, the Arctic and the Mediterranean. Leif Ericson, an Icelandic explorer, is regarded as the first European to have landed in North America, where a settlement was established in about 1000AD. Use of the longship for military purposes diminished during the 11th century as Viking raids became less successful following the rise of centralised regimes throughout Europe and the strengthening of coastal defences. However, longships remained in use until the 13th century, and their character and appearance were reflected in Scandinavian boatbuilding traditions until the early 20th century.

Through their extensive contact with other cultures, the Norsemen were a major influence on the shipbuilding technology of their day and following the Viking Age, progress in the design of the sailing ship was unprecedented. The cog, a vessel which first appeared in the 10th century as a relatively inexpensive cargo vessel for use predominantly in the Baltic, during the 13th century became the first European ship to use a rudder fixed permanently to the stern. Originally influenced in its design by the knaar, the cog evolved over three centuries from being a small undecked Baltic trader into a true seagoing ship, decked and capable of extending its range into the North Sea. Raised platforms were constructed fore and aft for defence against pirates and, as freeboard increased, the rudder was relocated to the stern, permanently attached using pintles and gudgeons. The transformation of the cog corresponded with the rise of new cities throughout Europe and an emerging merchant class. Traders, in the footsteps of Marco Polo of Venice, journeyed overland to the east and returned with new knowledge, including use of the magnetic compass for navigation and detailed descriptions of Chinese junks. The magnetic compass was quickly adopted and considered a necessity for navigation by the early 13th century, however the advantages of the Chinese lugsail were overlooked and the cog retained the square rig of the knaar. Despite stern-mounted rudders already being in use in China for more than a thousand years, historical evidence suggests that the cog's stern-mounted rudder was invented independently in northern Europe.

Struggles for power and territory throughout western Europe and the appearance of northern European ships in the Mediterranean during the 14th century provided the stimulus for subsequent advances in the design and construction of seagoing vessels. On the Iberian Peninsula, the amalgamation of elements of design adopted from the northern European ships and the traditional boats of the Mediterranean led to the emergence of Europe's first true ocean-going ships, the caravel and the carrack, used by the early Portuguese and Spanish explorers of the 15th and 16th centuries. Clinker-planked hulls had reached their structural limits in the largest cogs and, in the Mediterranean tradition, the caravel and carrack were carvel-planked. Also in the Mediterranean tradition, both vessels were fitted with two or more masts, the split rigs offering flexibility in sail handling and improved manoeuvrability. Caravels, the smaller of the two designs, varied in length from about 20 to 30 metres, were narrow with a beam of less than one-third of the overall length, and were fitted with a low forecastle and a minimal sterncastle. Usually carrying lateen sails on all masts, caravels were considered to be the handiest sailing ships of their day. Carracks, by design, were the beasts of burden, ocean-going transport ships able to undertake long voyages at sea independently of ports en-route. High forecastles and towering sterncastles offered space for the crew and protection from attack by small craft, while the stable deck allowed for the placement of cannons. As their sailing rigs evolved, carracks were generally fitted with three or four masts, usually square-rigged on the foremasts and mainmast and lateen-rigged on the mizzen.

The development of ocean-going ships heightened the prospect of direct trade with the East and led Europe into its Age of Exploration, initiated by Portugal during the 15th century, followed soon after by Spain. Through their Arab neighbours the Iberians had already gained some knowledge of the geography of Africa and Asia and set out to explore Africa's west coast in search of a route to India. Portugal, after decades of reconnaissance, during which time they improved navigational techniques and systematically edged further offshore to establish a viable sailing route, eventually reached the Cape of Good Hope in 1488. Spain, in a bold move to gain a competitive edge over Portugal and prompted by an Italian, Christopher Columbus, immediately organised a small fleet to sail the assumed direct route to the East Indies, westward across the Atlantic. Knowledge of a spherical Earth and its approximate diameter was widespread among European scholars, and most European navigators correctly deduced that a non-stop voyage from Europe to Asia was beyond the range of their ships. Portugal and England had each rejected earlier proposals from Columbus and so it was a Spanish fleet that undertook the famous trans-Atlantic voyage of 1492. The flagship for the expedition was the *Santa Maria*, a small carrack approximately 25 metres in length and carrying a crew of about forty-five men, accompanied by two caravels, the *Pinta* and the *Nina*, each about 20 metres in length. Whether by mistake or by brilliant deduction from accumulated knowledge, Columbus successfully sailed across the Atlantic and made his predicted landfall, inadvertently discovering the New World. Importantly, Columbus' return to Europe using a more northerly route established amongst European sailors the feasibility of ocean voyaging.

Portugal continued its thrust towards India and China via the Cape of Good Hope and by 1515 had established trade agreements, commissioning carracks to purchase cotton and spices from India and silk from China in exchange for Portuguese silver. Meanwhile, Spanish explorers, spurred on by the discovery of the New World, had crossed the Isthmus of Panama and sighted the Pacific Ocean. In 1519, Ferdinand Magellan, originally Portuguese, set out from Spain with a fleet of five carracks on an ambitious voyage to establish, if possible, a westward route to the East Indies. By negotiating an elusive passage at the southern extremity of the South American continent Magellan emerged into the vast Pacific Ocean towards the end of 1520, reaching the Philippines a few months later. Magellan was subsequently killed in battle and only one carrack, the *Victoria*, commanded by Juan Sebastian Elcano, completed the voyage, eventually returning to Spain via the Cape of Good Hope in 1522 to become the first ship to circumnavigate the globe.

Prior to the arrival of the first Portuguese and Spanish explorers into the Indian and Pacific

Oceans, the evolution of ocean-going ships in Asia had far surpassed that of Europe. Seafaring merchants, taking advantage of the seasonal monsoon winds, had been trading between India, Arabia, eastern Africa and the China Sea for centuries. By the beginning of the 15^{th} century, junks in China had developed into the largest and most seaworthy ships in the world and were massive by European standards. In 1405, almost a century before a Portuguese fleet of four ships first rounded the Cape of Good Hope en-route to India, Chinese junks, under the command of Admiral Zheng He, had embarked on the first of several Chinese naval expeditions to the Indian Ocean. China's intentions had been to establish a presence in the region and to impose control over trade by an impression of enormous strength. Zheng He's huge fleets consisted of up to three hundred junks crewed by an estimated twenty-eight thousand men, the largest vessels being more than 120 metres in length and rigged with as many as six masts. Between 1405 and 1433, the Chinese expeditions had sailed as far west as Africa and Arabia carrying gifts of gold, silver, silk and porcelain and returned with novelties such as zebras, giraffes, ostriches, ivory and camels. Although the Chinese had occasionally resorted to force during these expeditions, a more aggressive culture in the same dominant position could easily have conquered many of its rivals. Instead, less than a century after Zheng He's final expedition, all of China's large junks had been totally destroyed and China, during the Age of Sail, would not re-emerge as a naval power.

Throughout China's history, her governments had generally shown a disinterest in sea power. An extensive landmass, capable of sustaining large inland populations far from the sea, and geographical features that encouraged development in isolation from the remainder of the civilised world, directed the economic considerations of governments mostly towards inland activities. For centuries, international trade, particularly by sea, was considered to provide insufficient return to warrant expenditure. Until about the 8^{th} century most international maritime trade with China was conducted by Arab, Persian and Sinhalese vessels, while Chinese shipping activity was mostly focused on river craft, some of which were huge, having as many as five decks and carrying crews of several hundred people who lived their entire lives onboard. Although stern-mounted rudders had been in use in China from as early as the 1^{st} century and fore-and-aft junk rigs since the 3^{rd} century, it was not until about the 9^{th} century that ocean-going junks were developed. By the 13^{th} and 14^{th} centuries the largest ocean-going junks were carrying crews of up to three hundred sailors and were fitted with private cabins for as many as sixty merchants, trading with countries as far west as Egypt. The pinnacle of the evolution of Chinese ocean-going junks had been reached with Admiral Zheng He's naval expeditionary fleets of the early 15^{th} century, after which China entered an isolationist phase under a new emperor and a xenophobic government that dismantled the naval forces and discouraged involvement in international maritime trade. Incredibly, in 1525, shortly after Portuguese ships first entered the South China Sea, the Chinese government ordered the destruction of all large junks and declared it a crime in China to put to sea in a ship with more than two masts, inevitably resulting in the eventual loss of China's ocean-going technology.

Chinese junks, besides having had an advantage in size, were more advanced than their European counterparts in aspects of design and construction that were not immediately apparent to the Portuguese sailors. In construction, not visible externally, junk hulls were divided into compartments by transverse and longitudinal bulkheads, not only producing great structural rigidity but also offering protection against sinking. Despite their narrow beam, shallow draft and high freeboard, the Chinese vessels were also extremely seaworthy. In their rigging, junks had been multi-masted for centuries and the characteristic, fully-battened, fore-and-aft lugsail had reached a high degree of perfection, offering the possibility of true windward performance. Sail handling did not require the crew to go aloft, all of the routine tasks were capable of being carried out from the safety of the deck. Lines attached to the aft ends of the sail's battens controlled the sail shape and reefing was performed by simply easing the halyard and partly lowering the sail. To reduce leeway, in an attempt to take advantage of the windward capabilities of the rig, the lateral resistance of the shallow hulls was increased by fitting abnormally large rudders that could be raised in shallow water and, on some junks, by using leeboards, which had been in use in China from about the 8th century.

Remote from both Asia and Europe, a more effective method of using boards to reduce leeway had already evolved, albeit on a square-rigged raft that would otherwise be considered inappropriate for sailing, other than directly downwind. For hundreds of years before the arrival of Spanish explorers in the Pacific during the 16th century, deep-sea fishing and maritime trade had been vital components of the economic system of the Inca nation of South America. Remembered mostly for an empire that once extended high into the Andes Mountains, the Incas were also experienced mariners, fishing far offshore and regularly transporting goods and people by sea along the coastlines of Peru and Ecuador. Reed boats were the preferred choice of individual fishermen for inshore use off the desert coastline of Peru but for offshore work large wooden rafts were constructed using solid logs felled in the coastal forests of Ecuador. The largest rafts were capable of carrying as many as fifty people and were constructed from seven, nine or more balsa logs, lashed to one another with ropes and to other balsa logs acting as cross-beams, on top of which was built a platform to keep people and goods from getting wet. The bow was V-shaped, the stern squared off and fitted with a steering oar. Paddles were used for manoeuvring but the primary propulsion was provided by a square sail set on a bipod mast. To increase the lateral resistance of the shallow hull the Incas had cleverly contrived wooden centreboards that were inserted between the buoyant balsa logs fore and aft of the mast. As well as reducing leeway, the centreboards could be raised or lowered by the crew to alter the underwater profile of the raft while underway, balancing the forces of wind and water to assist the steering.

Inca rafts were first encountered by Europeans in 1526, off the coastline of northern Ecuador. After crossing the Isthmus of Panama and discovering the Pacific Ocean, Spain had established settlements on the west coast of Central America and began to build ships there for further exploration. A caravel, sailing south on a voyage of discovery down the Pacific coastline of South America with a crew of ten, made that first encounter, a north-bound balsa raft of similar size, carrying about twenty crew and passengers. Five more rafts, laden with cargo, were observed before reaching Peru. The Spaniards marvelled at the seamanship of the Incas, the buoyancy of their unusual vessels and the quality of their cotton sails. However, although the Inca rafts were comparable in performance to the Spanish vessels, the significance of the rafts' centreboards was initially overlooked by the Spaniards and would not be fully realised for another two centuries. In the meantime, Spain continued to explore the Pacific coastlines of South and North America and resumed the quest to establish a route to the East Indies by pushing westward across the Pacific. By the late 1500s, Spain had colonised the Philippines and ships sailing from the Americas had discovered several islands in Micronesia, the Solomon Islands in Melanesia and the Marquesas Islands in Polynesia, but the majority of the Pacific remained unexplored.

The Pacific Ocean had been named by Magellan when he first sighted that 'peaceful sea' in 1520, but the very existence of the Pacific had been a complete unknown to Europeans until 1513. For centuries scholars had theorised the probability of a great southern continent to counter the landmass of Europe and Asia, not an ocean that stretched halfway across the world from the coastline of Peru to the islands of Indonesia, covering a third of the Earth's surface and larger than the combined land area of the planet. Disproving the existence of the supposed 'Terra Australis Incognita', the 'Unknown Land of the South', required almost three centuries of further exploration by European ships of several nations, revealing instead a spattering of some twenty-five thousand islands, ranging in size from the smallest of coral atolls, barely rising above sea level, to the ancient island-continent of Australia. To the amazement of the Europeans, practically all of the habitable islands, including some that are extremely remote, were found to be occupied by stone-age people who possessed only simple sailing canoes and no navigational instruments. Doubtful of the capability of such 'primitive' people to discover and settle the more isolated islands, the question of how the Pacific happened to be populated mystified European scholars for centuries.

Human occupation of the Pacific, judging by modern evidence, began at least 65,000 years ago when sea levels dropped during the most recent Ice Age, providing a brief window of opportunity for overland migration through the Indo-Malaysian Archipelago towards the Australian continent. Australia, at the time joined to New Guinea and Tasmania in a single landmass, remained separated from the nearest shoreline of Indonesia by more than one hundred kilometres of open sea, implying that the first people to venture into the Pacific, whether intentionally or accidentally, already had the technical skills of accomplished seafarers. Those pioneer settlers brought with them languages that were fundamentally African and, in relative isolation, developed unique characteristics and technologies. In Australia the ancestor's of today's Aboriginal population spread southward to inhabit the entire continent, while in the north the migration through New Guinea extended eastward into the Pacific via the Solomon Islands, Vanuatu, New Caledonia and eventually Fiji, to occupy a region known today as Melanesia. The more distant islands of Polynesia, roughly contained within a triangle formed by Easter Island in the east, Hawaii in the north and New Zealand in the south were discovered and settled much later by a people whose ancestors are believed to have originated in the region of the South China Sea. Reaching the Philippines and New Guinea sometime between the 5th and 3rd millenniums BC, these seafarers progressed in small groups along the Melanesian archipelago until they too eventually reached Fiji. Archaeological evidence suggests that during the 2nd millennium BC, the migration resumed, heading eastward to the uninhabited islands of Tonga and Samoa where, in isolation, it is believed the physical traits, the language and culture of the Polynesians were developed. During the 1st millennium BC the Polynesians began to disperse throughout the Pacific, reaching the Marquesas, the oldest known occupied islands in Eastern Polynesia, by the time of Christ. Rapa Nui, or Easter Island, the most isolated inhabited island in the Pacific, is believed to have been settled before the 5th century AD, followed by the major outer settlements of Hawaii and Aotearoa, the Maori 'Land of the Long White Cloud', more commonly known today as New Zealand. By the end of the 1st millennium AD the occupation of Polynesia was complete. In like manner, the tiny islands of Micronesia, scattered over a vast area of the Pacific between the Philippines and Hawaii, are believed to have been partly discovered and settled during the 2nd millennium BC by seafarers from the Philippines and Indonesia. Later settlement by Melanesians, from the west and south, and Polynesians, from the east, resulted in a mixed culture throughout Micronesia, with each of the many island groups retaining distinct histories and customs.

The phenomenal maritime history of the Pacific was, of course, unknown to Europeans when Magellan's ships, sailing downwind from the Americas, crossed that ocean in 1521. After three months at sea without sighting land Magellan's fleet had entered Micronesia and made the first European landfall on an outer Pacific island, which was promptly dubbed 'The Island of Sails'. There, on Guam, amidst a sprinkling of tiny islands in the western Pacific, Magellan had encountered a genuine maritime community whose very existence had depended on the sea for thousands of years. The island's boats, which were an integral part of that existence and astonished Magellan's expedition, were the unique Micronesian outriggers, or proas, the fastest and most advanced sailing vessels in the entire world. Superficially resembling simple outrigger canoes which were variants of the dugout canoes that could be found in many 'primitive' cultures, Micronesian proas incorporated elements of design that were so progressive that they would not be fully understood by outsiders for centuries. To the Spanish explorers in the Pacific, the design, construction, handling and performance of the proas presented an almost total contradiction to the long-established theories and practices of European ship-builders and sailors of the 16^{th} century. The Micronesian proa's distinctive design characteristic, a narrow asymmetrical main hull which was always kept to leeward of the outrigger when under way, necessitated that the craft be able to sail with either end forward. Reversing course entailed the unconventional procedure of shifting the rig and the steering paddle from one end of the vessel to the other. Yet, despite these apparent disadvantages, most incredible was the unrivalled ability of the proa to sail faster than the wind. Refined over thousands of years of evolution and innovation, the more subtle features of these outwardly simple craft were beyond the immediate comprehension of the Europeans: the hydrodynamic lift to windward developed by the asymmetry of the main hull, which was flat to leeward and convex to windward; the main hull's intentional list to leeward, providing a vertical component of the lift and, consequently, reduced resistance at speed; the design of the outrigger, specifically devised to allow the solid windward float to move independently of the main hull over waves; and the aerodynamic efficiency of the powerful 'Oceanic' lateen sail, set on a mast which could be canted to windward in strong winds to reduce heeling and produce vertical lift in the same manner as a modern sailboard. Not until the unfolding of the science of flight during the 19^{th} and 20th centuries would the extraordinary creativity of the architects of the Micronesian proa begin to be truly appreciated.

For the European explorers, fortune-hunting in the Pacific, Micronesia was merely a featureless expanse of ocean dotted with small tropical islands, most of which were little more than coral outcrops. Land resources were few and the communities isolated, some separated from their neighbours by hundreds of kilometres of open sea. Coconut and pandanus trees provided most of the meagre timber supply for the construction of dwellings and boats, as well as the raw materials for weaving and cordage. Yet the Micronesians had existed within that formidable environment for more than two thousand years, dependent almost entirely on the sea for their precarious survival. The sheltered waters of the lagoons and the open seas beyond the protective reefs teemed with fish, while the reefs themselves bristled with shellfish and crabs. Fishing from boats, within the lagoons, around the reefs and offshore was, by necessity, the principal activity of the atoll communities. For the islanders of Micronesia, the sea was their life and it was within that co-existence with the land and the sea that the unique Micronesian proa had evolved.

Constructed in a range of sizes dependent on the resources at hand and the vessel's intended use, traditional Micronesian proas can nevertheless be broadly separated by length into three distinct groups. Most numerous were the canoe-like craft owned by individuals and used within the lagoons for fishing and transport. Usually paddled, these small proas were generally no more than about 5 metres in length and were designed to be single-handed. Often rigged with sail, they were also used for recreation, undoubtedly a most significant factor with regard to the evolution of the proa design generally. Sailing, as a sport, may not have been unique to Micronesia in the ancient world but the warm tropical waters, the constant trade winds and the proximity of the sheltered lagoons could not have provided a more enticing environment to an adventurous and fun-loving people whose lives revolved about boats and the sea. Early European accounts of

Micronesian life tell of the Micronesians' passion for their proas and their enthusiasm for racing. Challenges were immediately issued whenever a new boat was launched and sometimes the ensuing informal regattas would last for days until a particular boat was declared to be the winner. On other occasions formal regattas would be organised by a chief and invitations to compete sent to neighbouring villages. Competition plainly encouraged a contribution of ideas and experimentation leading to improvements that were applicable to larger craft, which were more demanding of resources, both in terms of materials and manpower. Among those larger proas, the mid-sized group, about 6 to 9 metres in length, were the workboats of Micronesia. Used for much the same purposes as the smaller canoe-like craft, the range of these more seaworthy proas extended offshore, not only beyond the reefs but often beyond the horizon. Capable of carrying as many as ten people in the larger sizes, these craft were also built for speed, a consistent feature throughout the entire size range, including the largest of all, the voyaging proas, some of which exceeded 30 metres in length. Designed primarily for transport between island groups, these extremely large craft were capable of carrying as many as fifty people and could comfortably maintain speeds in excess of 10 knots at sea. One proa, 15 metres in length and traditionally built towards the end of the 20th century, was recorded as sailing at speeds of up to 22 knots on a broad reach in 20 knots of wind, testimony to the ingenuity of the Micronesian boatbuilders of the past and the value of traditional knowledge accumulated over thousands of years.

Although a stone-age society without a written language, no less impressive than the design of their proas was the ability of the Micronesians to regularly sail their unique vessels across hundreds of kilometres of open sea without charts or instruments. Instead, the Micronesians made use of their intimate knowledge of the stars, the ocean swells, the flight patterns of birds and other natural signs to pilot their craft. Specially selected for their intellect, traditional navigators were trained from boyhood to have an extensive knowledge of astronomy and the ocean environment, to commit all of that knowledge to memory for future use and, when necessary, to be able to bring together all of the appropriate information to make an accurate landfall on a distant island. Typically, the extensive seafaring knowledge of the Micronesians was generally overlooked and ignored by the early European explorers and not until the 20th century were the Micronesian methods of navigation recorded in detail. In 1969, one of the few traditional navigators remaining within Micronesia demonstrated his expertise by accurately piloting a modern sailboat from Puluwat, in the Caroline Islands, to Saipan, in the Mariana Islands, and back, without instruments. About 700 kilometres of open sea separate the two islands and Hipour, the navigator, made use of memorised sailing directions handed down through generations, even though that particular voyage had not been made by his people for more than sixty years. Only then was the sophistication of the Micronesian navigation system finally realised by the outside world and the high regard in which navigators were held within Micronesian society more clearly understood.

Equally proficient in navigation were the Polynesians, who had used techniques similar to those of the Micronesian navigators to explore and settle the more widely scattered islands of the Pacific, an incredible feat not suspected by Europeans until the 18th century voyages of another wide-ranging and accomplished navigator, an Englishman, James Cook. In the interval between Magellan's east-west crossing of the Pacific in the early 16th century and Cook's expeditions, Europe had witnessed major changes in philosophy and science, instigated by renowned individuals such as Copernicus, Tycho Brahe, Galileo Galilei, Johannes Kepler, Rene Descartes, Blaise Pascal and Isaac Newton. By the end of the 17th century a scientific revolution had occurred within Europe and the growing body of mathematical and mechanical knowledge had quickly led to the invention of machines to replace animal and manual labour, resulting in the beginnings of the Industrial Revolution of the late 18th century. Capitalism, formal education and the increased availability of newspapers and books had undermined the influence of the traditional religious sources of authority, replaced by a move towards science and rational thought. As a consequence, by the mid-18th century Europe had entered a new era of oceanic exploration. Whereas the European explorers of the Pacific in the 16th and 17th centuries had been searching for trade routes to exploit the riches of Asia, European nations, particularly Britain and France, began to commission scientific expeditions primarily to acquire knowledge of the natural world. Cook, a British naval officer who had established his credentials as an outstanding hydrographer in the Atlantic, was chosen to command three such voyages into the Pacific, from 1768 until his untimely death in Hawaii in 1779. Accompanying Cook were scientists and artists to collect specimens of exotic plants and animals and to record the languages and customs of the inhabitants. Traversing the Pacific and Southern Oceans, far beyond the extremities of the Polynesian triangle, Cook's expeditions dispelled forever the myth of a vast southern continent but identified, for the first time, the cultural similarities of the widely dispersed islanders, leading to Cook's personal speculation of a Polynesian nation that had, on their own, explored and settled the Pacific from the west.

Canoes, to about 30 metres in length and lashed together in pairs by the use of connecting cross-beams, had been the primary voyaging craft of the Polynesians during their migration into the Pacific. Spaced apart to gain stability, the full-bodied hulls provided the capacity to carry heavy loads without sacrificing speed and decks built on the crossbeams delivered the required living space for the migrating families, their plants and animals. Comparatively few deep-sea double voyaging canoes remained in use at the time of Cook's voyages, the major Polynesian migrations having been completed almost 1000 years earlier. In the western island groups of Tonga and Samoa, and the neighbouring Melanesian island of Fiji, the double canoes had been largely superseded by variations of the Micronesian proa, recognised and accepted as being of superior design for inter-island travel. In New Zealand, double canoes were but a distant memory for a Maori population that was no longer dependent on the sea for survival. However, in the more remote island groups of east Polynesia, deep-sea voyaging canoes were still in regular use. Tahiti, which figured prominently in Cook's exploration of the Pacific, is recorded as having special double voyaging canoes and navigators at the time of the initial contact with Europeans. There, the Tahitian double canoe, or pahi, was characterised by its V-shaped hulls, which reduced pounding at sea and provided the lateral resistance necessary to minimise leeway. The hulls were built with solid keels, usually carved from the trunk of a single tree, and planked with short lengths of timber sewn together with fibre, a common method of construction for larger craft throughout the Pacific islands. Rigged with two masts and high aspect-ratio sails made from matting, a typical pahi of 15 metres in length was capable of speedily transporting twenty or more passengers and their supplies across open seas. James Cook was to personally experience the highly developed nautical technology of the Polynesians on his arrival in Tahiti in 1774, coinciding with a gathering of local craft that numbered an incredible three hundred and thirty canoes, including double war canoes between 15 and 27 metres in length, crewed in total by almost eight thousand men. Admiration of that single event no doubt impressed upon Cook not only the power and regional influence of Tahiti, but also the strong social structure of Polynesian society and, in particular, the Polynesians' unique oceanic view of the world, making them well adapted, from a seaman's perspective, for the exploration and colonisation of the Pacific.

Voyages to Polynesia and Micronesia by European sailing ships, from the time of Magellan's crossing of the Pacific in the 16th century, without exception, necessitated following a downwind course from the Americas towards Asia. Adverse wind patterns and currents prevented a return by the same route from west to east, the direction of the Polynesian and Micronesian migrations.

Compared with the indigenous sailing craft of the Pacific islands, the European ships were slow, unwieldy and inefficient, especially to windward, but, despite those disadvantages, their sheer bulk afforded the capacity to sustain crews for prolonged periods through all but the most severe sea and weather conditions. By the time of Cook's expeditions into the Pacific during the 18^{th} century an expanding knowledge of the planet's geography and improvements to hulls and rigs had given Europeans the capability of sailing to the corners of the earth. For ocean-going voyages the inefficient carracks had been replaced towards the end of the 16^{th} century by galleons, which were subsequently used for both trade and warfare by the nations of Europe until the early 18th century, ultimately evolving in form and general appearance into the ship-rigged vessels of Cook's era. Hulls had become lower, narrower and more stable, offering less resistance above and below the waterline, resulting in improved seaworthiness and performance. Development of rigs had led to the vertical division of the square sails for ease of handling, the addition of studding sails and staysails and the replacement of the lateen-rigged mizzen with a more easily handled gaff-rigged sail. Bowsprits and removable topmasts extended the rigs forward and upward. Meanwhile, for smaller vessels working the home waters of Europe, knowledge gleaned from other parts of the world had influenced traditional thoughts on design. Fore-and-aft rigs, leeboards, centreboards and catamarans had each been experimented with, with varying degrees of success. Science, too, was beginning to have an impact.

The application of scientific and engineering principles to sailing ship design and construction is considered to have emerged during the mid-18^{th} century, in Europe. Advances in mathematics and increased opportunities for higher education generally had, at last, created an environment in which the interminable longing of designers to predetermine the stability and handling characteristics of a ship prior to construction could be satisfied, at least in part. *Treatise of the Ship*, a monumental work written by a French mathematician, Pierre Bouguer, and published in 1746, is acclaimed as the first comprehensive scientific analysis of sailing ship design. Three decades later, in 1775, a Swedish ship designer, Fredrik Chapman, published *Treatise on Shipbuilding*, regarded as the pioneering work of modern naval architecture. Chapman was the world's first naval architect in the modern sense and ultimately transformed traditional 'rule of thumb' ship design and construction. As the 18^{th} century drew to a close, the ascent of science and engineering, augmented by the knowledge accrued from the thousands of years of the evolution of sailing craft from all parts of the globe, had brought ship design to a turning point in its history, with an unexpected outcome. In retrospect, 1769, the year of Cook's initial visit to Tahiti during the first of his three expeditions into the Pacific, had marked the beginning of the end for the sailing ship. In that year, a Scotsman, James Watt, patented a new and efficient steam engine, the fully developed version of which went into production in 1775, ushering in the Age of Steam and, as a consequence, the Industrial Revolution. Experiments in the application of steam technology to shipping soon followed and by the turn of the century small steamers had demonstrated their potential to revolutionise harbour and river transport. Within the first two decades of the 19^{th} century steam technology progressed to a level where it became relevant to coastal and trans-oceanic shipping. After more than five thousand years at the forefront of human achievement, the sailing ship was becoming obsolete. River boats were the first to abandon sail and as commercial enterprises, governments and academia pursued the new technology, the demise of the sailing ship for trade and warfare became inevitable.

A NEW BEGINNING

The 18th century has been described as a century of mind-boggling change in almost every aspect of human life, activated by the scientific revolution that had swept through Europe during the previous one hundred years or so. Although termed a revolution, the foundations of the upheaval in science during the 16th and 17th centuries had been shaped originally by the early Greek and Roman philosophers, elaborated by later Islamic scholars and further developed in the universities of Europe since medieval times. Unquestionably, the true revolution in science had been the implementation of Galileo Galilei's interpretation of the nature of the universe and the insistence of the Italian philosopher that the natural world operated according to mathematical principles. This new approach to nature in Europe, thinking of the universe as having a mechanical structure, soon had enquiring minds questioning all manner of things, ultimately resulting in a transformation of ideas, not only within the sciences but in everyday life, challenging traditional religious and political beliefs. Fundamental to the mathematical approach to science was, of course, the use of mathematics itself, which, until the adoption in Europe of Arabic numerals from about the 12th century, had previously been based on the Roman numeral system, consisting of numbers and letters in a clumsy subtractive relationship. An advantage of the Arabic system, which we still use today, was that calculations could be done more easily and much more quickly than before and, as the sciences evolved, so, too, did the mathematics. Algebra, calculus, logarithms, probability theory, the binary system of numbers, the vernier scale, the slide rule and the mechanical calculator are just a few of the mathematical break-throughs that supported the scholars of the 17th century. In Europe, by the beginning of the 18th century, the traditional world of old was becoming, instead, a world of science based on mathematics and experimental research.

Sailing ship design is complex and, at the beginning of the 18th century, European shipbuilding was still largely based on traditional evolutionary techniques, relying heavily on the skills of the designer-builder to interpret the performances of existing vessels and, through long experience with similar types of vessels, to devise appropriate improvements for new designs. Indirectly, the basics of modern theoretical naval architecture had already been established by the works of several brilliant scientists during the 17th century but scientific theories specifically for the design or behaviour of ships had not yet been advanced. Lines drawings to accurately depict the shape of a hull were only in the very early stages of development and design plans, if any, generally resembled the less demanding architectural drawings used for the construction of buildings. Although scientific methods were available to support some aspects of ship design, ship-builders generally considered the scientific input to be of little practical benefit, an opinion, incidentally, that would persist, particularly amongst small-boat designers and builders, until the coming of the computer age in the late 20th century. Typically, man has relied on his everyday experiences to understand the natural world in which he exists and for thousands of years, since the launching of the first primitive boat, nature has provided man with inspiration for improvement in the design of his water-craft. By observing aspects of nature, whether the shapes of fish, the flight of birds, the gentle movement of ripples on a pond or the pounding of ocean waves onto a shoreline, the boat-builders of our distant past, through intelligent thought and experimentation, had already developed the fundamentals of ship design. Although incapable of expressing themselves in modern scientific terms, traditional designer-builders throughout the world undoubtedly recognised that the total resistance to the forward motion of any vessel travelling on the water surface, regardless of the method of propulsion, is essentially composed of three distinct elements: form resistance, surface friction and wind resistance.

Form resistance is the inevitable result of a vessel having to push water aside as it proceeds across the water surface. For a conventional vessel, most of the form resistance is a consequence of the energy expended in the formation of the waves which are plainly visible in the vessel's wake but, to a lesser extent, additional form resistance is also caused by the less noticeable drag through solid water of any appendages, such as a rudder. Other than at very low speeds, wave-making alone is the major contributor to a vessel's total resistance, the magnitude of the forces involved being directly dependent on the size, the shape and the speed of the vessel. Friction, between the water and the wetted surface of the hull and its appendages, is another source of resistance, clearly evident to any sailor from the effort required to propel a vessel burdened by marine growth. Wind resistance, not unlike form resistance, is the consequence of a vessel having to push air aside as it proceeds through that medium and, given the obvious braking effect of, say, a square sail on a vessel being rowed through a calm, wind resistance has, without doubt, been common knowledge to sailors since before recorded history. From a practical viewpoint, surface friction and wind resistance can be reduced intuitively and, by and large, those aspects of hull design have never been given a great deal of consideration by designers. Although complex from a scientific point of view, surface friction can be decreased by simply smoothing the hull's wetted surface and wind resistance lessened to a large extent by minimising the exposed above-water area of the vessel, particularly the rig. In contrast, the amount of energy obviously wasted in creating excessive wake and the difficulty of understanding exactly the underlying principles of the wave-making process, have meant that throughout the ages the quest to refine the shapes of hulls to minimise form resistance and thereby reduce the driving force required for a vessel's propulsion, has led to that specific aspect of hull design to be pondered and investigated intensely, in fact, almost exclusively.

Wave-making can be reduced by merely sharpening the bow of a vessel, a primitive method used by the Incas on their balsa-log rafts built for plying the Pacific coastline of South America. Further refinement of traditional hull designs by trial and error, without the aid of scientific knowledge, has, however, proven to be hugely successful, indubitably demonstrated by the extraordinary sailing performance of the Micronesian proas encountered by Ferdinand Magellan in the early 16th century. Full-scale experimentation, though, does have limitations, particularly for ships. As the size of vessels increases, construction rapidly becomes more costly in terms of resources and labour, making variations in design more of a risk and therefore less likely. High costs also result in large vessels being built in fewer numbers, further reducing the opportunities for their methodical improvement. An obvious alternative to full-size experimentation is the use of scale models, a widespread practice in the traditional design process, but models, too, have limitations. Accurately interpreting a model's performance and correctly assessing how any knowledge gained might be applicable to a full-size vessel is not straightforward. Inevitably, during the 18th century, not only because of the size of the ships being built but also because of the pressures of an increasing control over naval matters by national governments and their growing bureaucracies, European ship-builders began looking toward the expanding influence of mathematics and the natural sciences to facilitate a new beginning in the ship design process.

Archimedes, in the 3rd century BC, had been the first to describe the physical principles of the buoyancy and stability of floating objects at rest in still water, instigating the science of ship hydrostatics, but almost two thousand years were to pass before adequate mathematical tools were developed to enable Archimedes' theories to be extended beyond their fundamental concepts. The principal architect of the branch of modern physics that encompasses ship design theory was Isaac Newton, born in England in 1643, about the time of Galileo's death. *Mathematical Principles of Natural Philosophy*, written by Newton and published in 1687, is one of the foremost scientific works of all time, earning Newton the reputation of being one of the most influential people in human history. Encapsulating the scientific theories of his era, in his writings Newton expressed, amongst other things, his personal interpretations of force, mass, gravity and the famous three laws of motion which dominated the science of physics until challenged by the German-born Albert Einstein's famous general theory of relativity in the 20^{th} century. So advanced were Newton's ideas that his contemporaries were initially stunned by the publication of his works, but by the beginning of the 18^{th} century Newton's principles of 'classical mechanics' were being applied and the foundations were being laid, at last, for the mathematical investigation of fluids in motion and the forces experienced by vessels underway.

Resistance to the forward motion of an object moving through a fluid has been the subject of much scholarly thought since ancient times, particularly in relation to ballistics, the study of the motion of projectiles through air. Newton, experimenting with a pendulum swinging in air and in liquid, concluded that the magnitude of the force that was diminishing the pendulum's motion, the resistance caused by the fluid, was proportional to the square of the velocity of the pendulum. Similar results had also been reached independently by other scientists measuring, instead, the force required to either tow an object in a tank or hold an object stationary in a flowing stream. For the first time in history, the form resistance of an object moving through a fluid could be expressed mathematically:

$$Resistance \propto v^2$$

Newton's experiments were directed towards objects moving completely immersed in a fluid, as opposed to an object travelling on the fluid surface and generating surface waves as a consequence. For simplicity, in advancing his explanation of resistance Newton intentionally ignored the effects of the viscosity of the fluid, the sluggishness caused by the cohesion of the fluid particles. Instead, for theoretical purposes only, Newton assumed the fluid to be a 'rare medium', a hypothetical, perfect, incompressible, elastic fluid comprised of particles of equal size and mass which are free to move without attraction between each other and behave in such a way that the laws of motion could be applied. In reality, Newton was aware that it was highly unlikely that any such fluid could exist in nature and that water, as free flowing as it may seem, has viscosity. Newton's explanation of the resistance experienced by an object moving through the perfect fluid was uncomplicated, basically suggesting that, as an object progresses, the forebody simply collides with the fluid particles in the object's path and those particles are then reflected as in any normal elastic collision. Extending that logic, Newton reasoned that the afterbody and the sides of the object, if parallel-sided, do not have any impact on the fluid particles and so do not contribute to the form resistance. According to Newton, the resistance is, therefore, the sum total of the impact forces over the frontal area of the object in the direction of travel, adding another dimension to the original formula, the area of the maximum cross-section:

$$Resistance \propto A\, v^2$$

Newton went further, reasoning that the unknown coefficient in the formula for resistance should remain constant for objects of similar proportions, regardless of their size, and that there ought to exist a 'solid of least resistance', a uniquely shaped object that, when moving through a fluid, would experience a minimum of resistance.

Publication of Newton's theories created a spate of activity by mathematicians who set about testing the validity of his ideas, the concept of the 'solid of least resistance' being particularly alluring for those delving into the performance of sailing ships. Newton had already determined, by calculation, his own solution for the most efficient form for a projectile in air, an object which could readily be described today as 'bullet-shaped', being cylindrical with a rounded, snub-nosed forebody. Direct application of Newton's relatively simple theories of resistance to ship design, however, proved to be disappointingly unsuccessful, not only because the physical properties of the real fluid were completely disregarded by over-zealous mathematicians, so too were the effects caused by the creation of surface waves, a phenomenon not experienced by the deeply immersed objects to which Newton was referring. In the practical world of shipbuilding, Newton's concepts were unconvincing. To the traditionalists, any hint of a suggestion that the forward sections of a vessel underway might propel water particles forward would have seemed ludicrous except, perhaps, for a small amount of spray generated at the bow. Equally, for the afterbody, the ancient practice of improving the performance of a vessel by shifting cargo forward to clear an immersed stern would have seemed an obvious indication that a hull's after sections also have a profound effect on a vessel's resistance, contradicting Newton's ideas completely. In Newton's defence, the application of his theories of resistance to a vessel moving across the surface of a real fluid was well outside the parameters that he had originally set.

From the viewpoint of the traditional ship-builders, Newton's simple formula for resistance would hardly have been a revelation. Newton's claim that the magnitude of the form resistance is directly proportional to the square of the velocity was not altogether surprising and simply reinforced common knowledge, that the higher the speed of a vessel the more difficult it was to gain an increase in speed. Application of Newton's formula could justify why all vessels ultimately reached a limiting speed which seemed impossible to exceed without a massive driving force, but the formula did not clarify why that upper speed limit depended on the length of the vessel, favouring longer waterlines for higher speeds. Likewise, the other component of Newton's formula, the resistance being directly proportional to the area of the maximum cross-section, was even less enlightening. The entire maritime world had already determined that long, slim hulls are more easily propelled than shorter, fatter hulls of similar volume. At the end of the day, Newton's theories had not provided the traditional designer-builders with any new concepts of practical use and, as a consequence, Newton's actual method of approach to resolve the problem of form resistance was bound to be queried as well. In their own minds, the designers of all types of conventional water-craft, then and now, tend to visualise the underwater surface of a hull in its entirety, held stationary while the water flows smoothly around the hull's curves. The designers' view is more akin to the flow of a stream around an obstacle rather than Newton's elementary concept of a moving object simply pushing individual water particles aside, except, perhaps, at the forward extremity of the waterline. Despite a growing distrust of Newton's theories of resistance, however, no markedly improved concepts for the resistance of ships were put forward for more than half a century until, in somewhat of a triumph for tradition, the 'streamlines' embedded within the imaginations of the designer-builders were able to be described more satisfactorily by a new generation of mathematicians.

A fresh, innovative approach to the study of fluids in motion was unleashed in 1727 by a young Daniel Bernoulli, destined to become a prominent physicist of the 18th century. Born in the Netherlands in 1700 into a Swiss family which was already well-established and renowned in the field of mathematics and its practical application, Daniel Bernoulli was initially trained as a medical doctor, although his enduring passion was to follow in the footsteps of his already-famous father, Johaan. At age 25, Daniel Bernoulli accepted an invitation to become a professor of mathematics in St Petersburg, Russia, assisted by a close friend, Leonhard Euler, seven years his junior and recommended for the position by Johaan, under whom Euler had been studying. Euler was exceptional, having previously graduated from the University of Basel as a Master of

Philosophy at age 16. While in St Petersburg, Daniel Bernoulli began investigating the flow of fluids, with a particular interest in the course of blood through the body, especially the relationship between blood pressure and the speed of the blood flow through the arteries. Together, Daniel Bernoulli and Euler experimented and, by inserting a small diameter, open-ended tube into the wall of a pipe, deduced that the height to which the fluid in the pipe rose up the tube was related to the velocity of the fluid through the pipe. Surprisingly, the faster the flow of the fluid through the pipe, the lower the height in the tube, suggesting that the faster the fluid flowed, the less pressure it exerted on the walls of the pipe. Publication of the results of this experiment soon had doctors inserting pointed glass tubes directly into patients' arteries to measure blood pressure. Intuitively applying the principles of the conservation of energy, Daniel Bernoulli took his discovery further, developing a completely original theory for steady, one-dimensional flow through pipes, leading eventually to the now famous Bernouilli equation, used to determine fluid velocities by pressure measurements. Daniel Bernoulli's experiments marked the beginning of a comprehensive reanalysis of fluid mechanics or, using the term invented by Daniel Bernoulli himself, the beginning of a new branch of physical science, hydrodynamics.

Within a decade of Daniel Bernoulli's ground-breaking discovery that the forces exerted by a fluid on the walls of a pipe decreased as the velocity of the flow increased, Newton's simple theories of the resistance of an object moving through a fluid were beginning to show their limitations. On-going scrutiny had continued to reveal fundamental discrepancies between Newton's concepts and the results being obtained by experiment. Inevitably, Newton's original hypothesis of individual fluid particles colliding with and being reflected by a moving object was eventually replaced by the 'new' concept of streamlines that trace out the path of the fluid particles. In principle, the streamline theory assumes that each of the fluid particles passing a given point in the stream follows exactly the same path and if the flow is to remain steady, without eddies, no two streamlines can ever cross. At locations where the stream becomes constricted, the streamlines move closer together and the velocity of the fluid particles necessarily increases, and vice versa. Somewhat unexpectedly, experiments also revealed that in a steady flow streamlines were deflected before reaching an obstacle and that individual fluid particles were not reflected during contact as Newton had imagined. Ultimately, Newton's relatively uncomplicated approach was abandoned and in its place Daniel Bernoulli's theories of fluid flow through pipes were ingeniously extended to encompass the steady flow of fluids generally by adopting the use of 'stream tubes' within a fluid, the tubes being made up of bundles of streamlines and behaving somewhat like pipes. The use of tubes of flow within the streamline model allowed the introduction of the concept of internal fluid pressure, thereby enabling a mathematical approach to the motion of fluids. However, as Pierre Bouguer's comprehensive *Treatise of the Ship* confirms, despite the radical developments taking place in hydrodynamics, by the mid-18th century no viable alternative for the calculation of the resistance of ships existed, other than those based on Newton's original hypotheses.

Daniel Bernoulli and Leonhard Euler, after returning from Russia, remained friends and often collaborated on each other's work. Euler, in particular, produced many outstanding contributions to the mathematical and physical sciences throughout his lifetime and, today, is recognised as one of the most extraordinary intellects of all time. Solid mechanics, astronomy and optics each gained from Euler's input but it was in pure and applied mathematics that Euler excelled and in fluid mechanics that he had a most significant impact, producing more than forty books or papers. A prolific writer, Euler generated a seemingly inexhaustible torrent of articles, particularly on mathematics, but generally less known are Euler's contributions to ship design, not only on the issue of resistance but also less mainstream topics such as the masting of ships, their stability, their motion at sea and investigations into forms of propulsion other than sail. On one occasion, Euler's learned ally, Daniel Bernoulli, entered, and won, a contest for the best design for a ship's anchor! Euler, like Bernoulli, was Swiss and the question arises as to why two men, one a brilliant mathematician and the other a renowned physicist, sharing a common heritage associated with a land-locked, alpine country and each having little or no practical experience of a maritime environment, should be remotely interested in the relatively mundane problems of the design and performance of sailing ships. One possible explanation is that during the lifetime of these two men, the deployment of sailing ships was clearly determining the future of Europe, if not the world. Navies were being formed, new lands were being discovered, fortunes were being made and wars won or lost at sea. But, in all probability, to men of Euler's and Bernoulli's disposition those factors may have been incidental, the sailing ship merely representing, to them, a complex hydrodynamic and aerodynamic muddle, the ultimate platform for the application of their personal lifelong passion, mathematics.

The characteristic that set Euler apart from many of his predecessors was his commitment to the methodical application of mathematical analysis to expose the fundamentals of scientific knowledge. Euler's early approach to the problem of a ship's resistance, for example, was to mathematically generate families of shapes that could be varied systematically to cover a whole range of hull forms, a method that, today, is regarded as standard practice. During the 1750's Euler, after years of mathematical analysis, finally rejected Newton's theories of resistance and established, instead, the 'continuum theory of fluid mechanics' based on Newton's principles of dynamics and their application to the newly discovered relationship between fluid velocity and pressure. Instead of attempting to specify the path of each fluid particle as proposed by Newton, Euler's ambitious undertaking was to specify the velocity and the density of a volume element of the fluid at each point in space at each instant of time. Euler developed his theories in a general form and derived complex mathematical equations for fluid motion, leading to a new level of understanding of flow phenomena by physicists. For the determination of the resistance of an arbitrarily-shaped object moving through a fluid, however, Euler could only outline the methodology, lacking the analytical tools to take the matter any further. Euler stipulated that first, the location of the streamlines over the body surface needed to be defined, then the velocities of the streamlines determined at every point along their paths to allow the calculation of the pressure at each of those points and then, finally, those individual pressures needed to be summed to determine the resultant force on the body. Although Euler may have been unable to solve the problem of form resistance within his own lifetime, Euler's approach to the mathematical analysis of fluids in motion was to become the fundamental influence of the on-going study of fluid dynamics.

Euler's revelations about fluid flow and his opinions of the difficulty of calculating the form resistance of an object moving through a fluid were a disappointment to ship designers. Seemingly, no sooner had science intervened in the design of sailing ships with the assumed potential to effect fundamental improvements to hull shapes and rigs when, because of the complexity of the issues involved, the fledgling sciences of hydrodynamics and aerodynamics were abrubtly elevated to the sole province of the elite mathematicians. Even worse, a mathematical solution to the all-important question of form resistance was considered unlikely in the foreseeable future. Rather than the uncompromising nitty-gritty that was initially expected from the sciences, mathematicians were now only capable of providing ship designers with little more than concepts for more efficient hull and sail shapes, about which the traditional designer-builders already had considerable practical knowledge. As a consequence, it is not surprising that the late 18th century witnessed the emergence of a new role in the shipbuilding industry, that of the modern naval architect, a ship designer with more than a modicum of mathematical knowledge, capable of bridging the division between the theoretical world of science and the practical world of ship design and construction. Fredrik Chapman, the author of *Treatise on Shipbuilding*, was one such person.

Chapman was born within the grounds of the Royal Dockyard in Sweden in 1721, the son of a former English naval officer who had moved to Sweden in 1715 and was captain of the dockyard. As well, Chapman's mother was the daughter of an English shipwright and so, unlike academics such as Bernoulli and Euler, ships and shipbuilding were very much a part of Chapman's upbringing. At age 15 Chapman went to sea, returning a few years later to work as a shipwright but his particular passion, acquired at a very early age while growing up in the dockyard environment, was ship design. Realising that many of the problems encountered in sailing ship design could be lessened by the application of mathematics and physics, Chapman moved to London in 1750 to study, returning to Sweden seven years later. With a practical understanding of ship construction and handling, he was able to effectively modify the theoretical knowledge gained in England to devise the means by which a ship's displacement and stability could be confidently calculated during the design process. But, like many of his contemporaries, Chapman remained unsatisfied with the state of the art of ship design. From his own experience at sea, he had already deduced that wave-making was an obvious and significant factor in the determination of the resistance of a ship and, while returning from London, had visited France with the intention of becoming more familiar with the published works of Bouguer and Euler. To Chapman, the complete omission of the wave-making process from ship design theory was likely to have been the main reason why a realistic concept for the resistance of ships had not yet been developed. Mathematicians and scientists, he realised, were more engrossed in developing a general theory for the resistance of wholly immersed objects, rather than specific solutions for objects such as ships moving across the fluid surface. To gain an independent insight into the wave-making process, Chapman then did what traditional designer-builders had always done, he resorted to the use of models.

Experimentation was seen by many ship designers as the only way forward following the disappointing failure of ship design theory. After Euler, models of all shapes and sizes were towed across ponds or in specially constructed tanks by various research groups throughout Europe to gather experimental evidence. Some models were exact representations of ship designs, aimed at determining the relative merits of similarly shaped hulls, while others were just simple ship-like shapes, to clarify the fundamental issues of hull design theory. In all of the experiments, the methods of determining the resistance were similar, generally employing a falling weight attached to a line to haul the model across the water surface. Measurement of the weight and the time taken for the model to travel a fixed distance, often fifty metres or more, allowed a direct comparison between the resistance of the model and its calculated velocity. The significant feature that differentiated Chapman's experiments from those of other researchers was that he first developed families of model shapes, as Euler had previously done for mathematical analysis, and then proceeded to conduct tests systematically, varying the location of the maximum cross-section and the shapes of the forebody and afterbody. From his experiments Chapman concluded that not only the shape of the forebody but also that of the afterbody and the location and area of the maximum cross-section contributed to the total resistance. Each of those factors was considered to be an integral component of the wave-making process. Somewhat surprisingly, tank testing towards the end of the century by an English experimentalist, Mark Beaufoy, revealed that friction, which was previously thought to have a negligible effect on the overall resistance of a smooth hull, was also another significant contributor to a ship's total resistance.

Taken as a whole, throughout the 18^{th} century substantial progress had been made. Euler, in particular, had established a solid foundation for the science of hydrodynamics and model research had ascertained that the total resistance of a ship's hull could effectively be separated into three basic components: the forebody resistance, responsible for the creation of the bow wave system, surface friction over the wetted surface of the hull and, finally, the drag of the afterbody. Yet, despite the intensive mathematical analysis of the first half of the 18^{th} century, followed by widespread tank testing during the remainder, more than a hundred years of rigorous investigation since Newton had failed to provide a useful solution to the question of a ship's resistance. Hydrodynamic theory and practical experiment had proven to be ineffective working independently of each other and the lack of appropriate theoretical knowledge had prevented the data acquired from model testing to be directy applied to the design of full-sized ships. Interestingly, notwithstanding the unanimous rejection of Newton's original interpretation of resistance, Newton's pronouncement that form resistance was proportional to the square of the velocity had been confirmed. At the close of the 18^{th} century, for the theorists and experimentalists alike, if further progress was to be made in ship design theory, the time had come to acquire an understanding of waves and the wave-making process.

THE TROCHOIDAL WAVE THEORY

The mathematics of surface wave motion in a real world is extremely complex. Viscosity, the depth and expanse of water and the nature of shorelines are just a handful of the many variables which confront the mathematician attempting to solve problems related to ocean waves, tides, tsunamis or relatively minor surface disturbances such as the wakes of ships. In the modern world, everyday mathematical problems, no matter how difficult, are popularly assumed to be easily resolved by computers, usually within seconds, if not instantly, but in the latter part of the 18th century, the early decades of the Age of Steam, the task of defining the complexities of surface wave motion by mathematics was almost inconceivable, not only to laymen but also to most scholars. The first known attempt to derive mathematical solutions for the behaviour of waves on a water surface is attributed to Isaac Newton, in the late 17th century. Newton, by experiment, correctly deduced that the velocities of waves in deep water are proportional to the square-root of their wavelengths but knew that his results were approximate, having been based on the assumption that the water particles within waves rise and fall vertically while knowing, from observation, that they actually follow an orbital path. Almost a century would pass before the complexities of surface wave motion were investigated more thoroughly, by two French mathematicians in the 1770's, Pierre-Simon Laplace and Joseph-Louis Lagrange. Working independently and directing their individual efforts towards different aspects of wave motion, Laplace and Lagrange each made considerable advances in water wave theory, clearing the way for subsequent works by other mathematicians, works that are still being perfected today.

In 1802 a Czech mathematician, František Josef Gerstner, offered a remarkably uncomplicated wave solution that not only avoided the application of higher mathematics but could be demonstrated by using graphical methods alone. Gerstner's wave, as it came to be known, was based on an assumption of a perfect, non-viscous, incompressible, elastic fluid of infinite depth and extent, as Newton had assumed for his theories of resistance. That assumption had already proven to be so far removed from reality that Gerstner's wave theory was disregarded by the academic community. Nevertheless, because of the theory's relative simplicity and a predilection for graphical solutions in a world before computers, Gerstner's wave did find acceptance amongst some in the more practical professions, particularly oceanography and ship design.

During the period in which Laplace and Lagrange were attempting, but failing, to resolve the complex mathematics of surface wave motion, Gerstner had been a student of mathematics and astronomy in Prague. In 1781, aged 25, Gerstner moved to Vienna, Austria, where he initially worked as an astronomer and, eight years later, was appointed professor of mathematics. *Theory of Waves*, one of Gerstner's most important printed works, was published in 1804. Soon after, Gerstner returned to Prague to take up a position of director of a polytechnic which he had helped to establish and was also appointed as a professor of mechanics and hydraulics, teaching there until his retirement in 1823. Throughout his distinguished career Gerstner had been more than a teacher, he was an accomplished mathematician firmly engaged in his nation's practical application of scientific knowledge, a seemingly altruistic outlook that, no doubt, had inspired his early presentation of the *Trochoidal Wave Theory* at a time when a complete, more advanced mathematical interpretation of surface wave motion simply did not exist.

The trochoid and its close relative, the cycloid, are geometrical curves that were already well known to the mathematicians of Gerstner's time, their existence having been identified by scholars from at least the early Greek era. Combinations of both linear and circular motion, the curves are those formed by the paths of points on a wheel rolling along a straight line. The cycloid, given its name by Galileo in 1599 and formed when the point tracing the curve is on the rim of the wheel, had already been subjected to intense scrutiny by mathematicians throughout the 17th and 18th centuries and was known to have many interesting properties. As early as 1659, the Dutch mathematician and physicist, Christiaan Huygens, had conceived the idea of a cycloidal pendulum, leading to his invention of the pendulum clock. By placing cycloidal shaped cheeks against the top of the string of a simple pendulum Huygens was able to demonstrate, in theory and in practice, that the bob traced a cycloidal path and that, unlike a simple pendulum, its period was independent of the amplitude of the swing. Huygens' invention was a breakthrough in time-keeping but undoubtedly the most famous application of the cycloid was the solution to a problem posed to his contemporaries by the Swiss mathematician, Johann Bernoulli, in 1696. The problem was to define the path that would allow a particle starting from rest to travel from a given point to a lower point in the shortest possible time, under the force of gravity alone. Several mathematicians successfully solved the problem, including Newton, who took less than a day. At first thought a straight line seems a reasonable solution, but allowing the particle to fall vertically at first to gain velocity would also seem to be an advantage. In reality, the solution was a cycloidal curve joining the two points. Surprisingly, the time taken for the particle to travel from one point to the other over the full length of the curve was found to be exactly the same time taken for a particle starting from rest at any intermediate point along the curve. It is little wonder that Gerstner, aware of the remarkable properties of the cycloid and knowing that the curve is not only related to but is, in fact, a special case of the more general trochoidal curve, investigated the trochoid to elucidate the principles of surface wave motion.

The following explanation of the trochoidal curve and, in particular, the derivation of the mathematical equations pertaining to the *Trochoidal Wave Theory* have been adapted, principally, from the book *Theoretical Naval Architecture* written by Edward L. Attwood and published in 1922 by Longmans, Green and Co., London. Attwood was a former member of the British navy's Corps of Naval Constructors and lecturer on naval architecture at the Royal Naval College, Greenwich, England. Attwood's book, *Theoretical Naval Architecture,* was an expansion of one of his previous works, *Textbook of Theoretical Naval Architecture*, first published in 1899.

The Trochoidal Wave

A trochoidal curve is the path traced out by a fixed point on a wheel as the wheel rolls, without slipping, in a straight line along a flat surface.

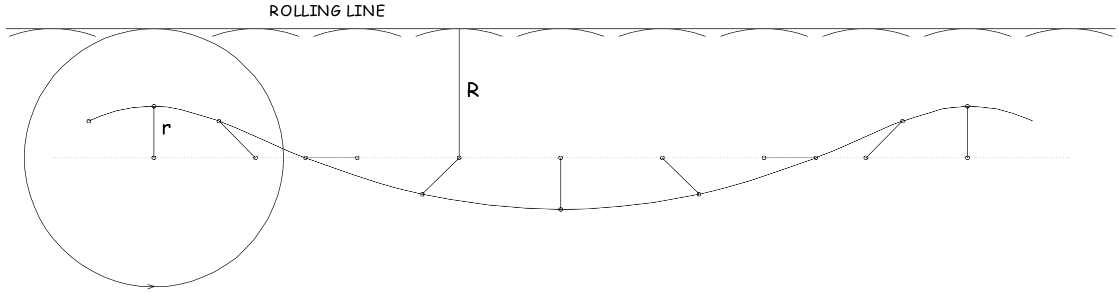


Figure 3-1

Figure 3-1 shows a typical curve traced by such a point, except that contrary to the usual concept of a rolling wheel, in this particular example the horizontal rolling surface is above, instead of below, the wheel, which is rolling from left to right.

As the wheel rolls it rotates in an anti-clockwise direction and points located at different positions on the wheel trace out unique paths, as shown in Figure 3-2. Although the paths vary in shape, each is a trochoidal curve that can be defined by the general equation of the trochoid.

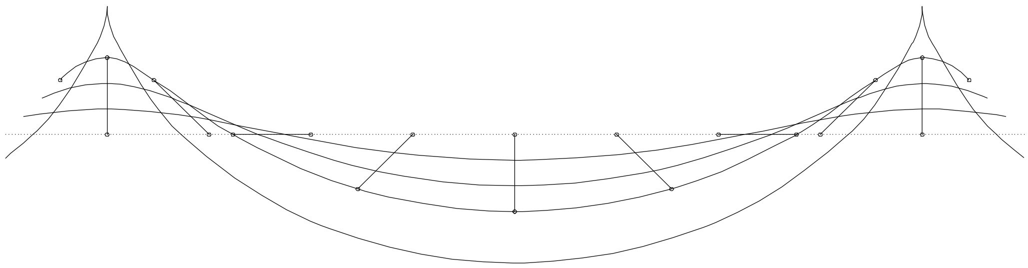


Figure 3-2

To determine that general equation, in Figure 3-3 the point O, at the 'crest' of one randomly selected curve, is adopted as the origin of the x and y axes. R is the radius of the rolling wheel and r is the radial distance from C, the centre of the wheel, to S, the point on the wheel tracing out the curve. θ is the angle of rotation of the wheel as it moves from left to right and rotates anti-clockwise, as shown in Figure 3-1. At P the wheel has travelled a distance of Rθ from its original position, Rθ being the length of arc on the perimeter of the wheel that has come into contact with the rolling surface as the wheel rotated through the angle of rotation, θ.

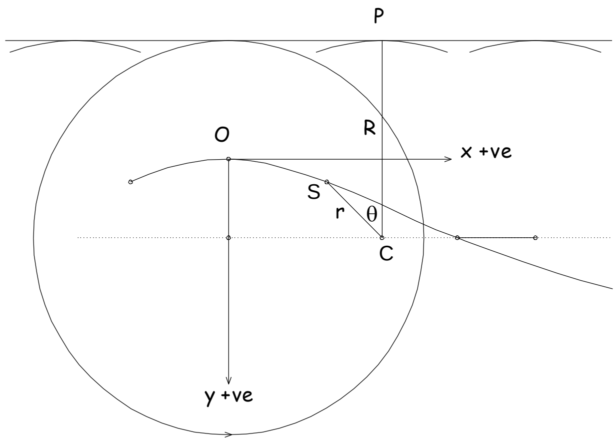


Figure 3-3

By inspection, the coordinates of any point S on that particular curve are given by;

$$x = R\theta - r sin\theta$$

$$y = r - r cos\theta$$

Together, these two equations define the trochoid. The x and y values of the point S cannot be expressed directly in terms of each other but are nonetheless directly linked through the values of R, r and θ.

For the trochoidal paths traced out in Figure 3-2, the variation in each case is in the value of r. As $r \to 0$ the amplitude of the trochoidal curve decreases until ultimately, at r=0, the point tracing out the curve is at the centre of the wheel, resulting in a path that is a straight line. Similarly, as $r \to R$ the amplitude of the trochoidal curve increases until, at r=R, the point tracing out the curve is on the perimeter of the wheel, resulting in a special trochoidal curve known as a cycloid.

Although trochoidal curves are wave-like in appearance, the concept used to determine those curves does not, in itself, provide a realistic representation of an actual wave formation. To turn the motion of a point on a revolving wheel into a wave formation on a water surface it is necessary to envisage innumerable such points, or, more realistically, innumerable water particles, each revolving in circular orbits about fixed centres. In effect, the rolling wheel in Figure 3-1 is replaced at each of its locations by a disc, revolving anti-clockwise about a stationary centre. The water particles at points such as S, always on the surface and all revolving with the same constant angular velocity, will then produce a wave formation, moving from right to left.

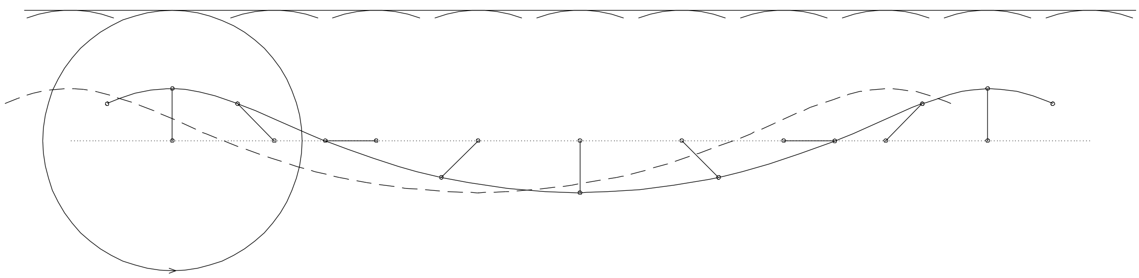


Figure 3-4

If Figure 3-4 is considered to represent a wave formation and not simply a trochoidal curve, from the rotation of the radial arms it can be deduced that on the crest of the wave each of the water particles has a component of its movement in the direction of the wave advance. In the trough, the reverse occurs and the water particles are moving in opposition to the wave advance. This particular characteristic of surface waves, being partly longitudinal and partly transverse, is easily confirmed by casual observation of the back-and-forth and up-and-down motion of a floating object, moving forward on the crests and backward in the troughs. The motion of the water particles also explains, in part, why vessels tend to surge forward on the crests of waves and are retarded in the troughs.

For particles below the water surface, the *Trochoidal Wave Theory* suggests that sub-trochoids exist, having the same wavelengths as the surface wave, with the crests and troughs in the same vertical line. The variations are in the locations of the rolling lines, which are progressively lower, and in the lengths of the radii r, which diminish with depth, becoming negligible at a depth equal to the wavelength.

The Trochoidal Wave and Still Water Level

The locus of the centre of the rolling wheel in Figure 3-1 is a straight line referred to as the 'line of orbit centres', parallel to the rolling line.

Trochoidal curves are asymmetrical about the line of orbit centres, being steeper in the 'crests' than in the 'troughs' when the rolling surface is above the wheel. The asymmetry is a characteristic particularly noticeable as $r \rightarrow R$ and, ultimately, at $r=R$, resulting in a cycloid, as shown in Figure 3-5.

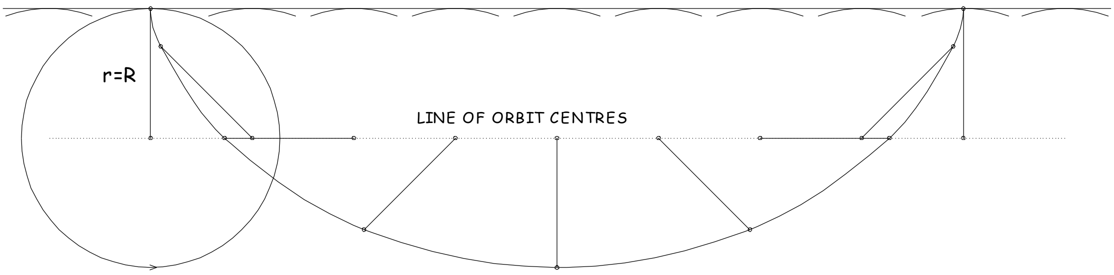


Figure 3-5

For a wave travelling across the water surface it would generally be expected that the volume of water in the crests matches exactly the volume displaced from the troughs. A wave surface of similar shape to that shown in Figure 3-5 clearly shows less volume in the sections of the wave above the line of orbit centres than in the trough below that line, indicating that the line of orbit centres is not the 'still water level' that existed before the creation of the wave.

Figure 3-6 shows the relationship between the line of orbit centres and still water level for a typical trochoidal wave.

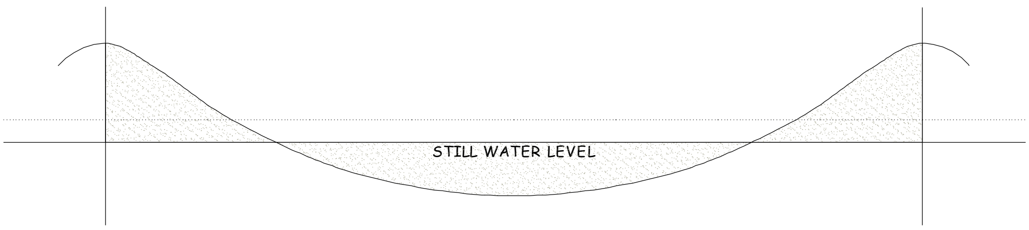


Figure 3-6

The line of orbit centres is always located above still water level, at a distance dependent on the relationship between r and R. As $r \rightarrow 0$ the amplitude of the trochoidal wave, 2r, is minimal and the line of orbit centres is practically at still water level. Conversely, as r increases, the height of the line of orbit centres above still water level also increases. For each unique trochoidal wave, the exact height of the line of orbit centres above still water level can be calculated.

The shaded area in Figure 3-7 shows the volume of water which is subject to possible displacement by the wave, between still water level and the line of the troughs. As the wave progresses, a volume of water equivalent to that removed from the trough is raised into the crests, as shown by the shaded area in Figure 3-8.

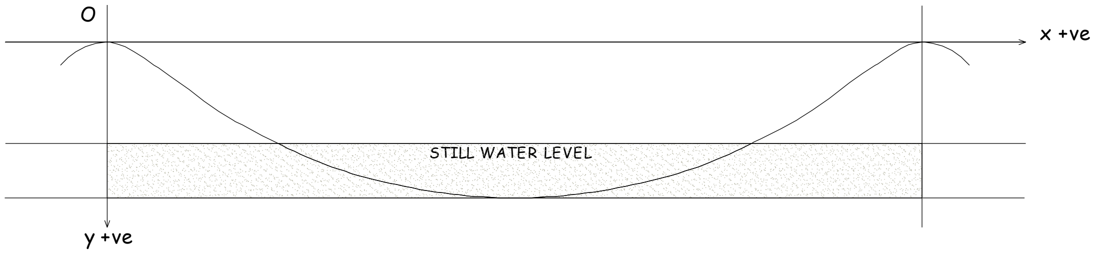


Figure 3-7

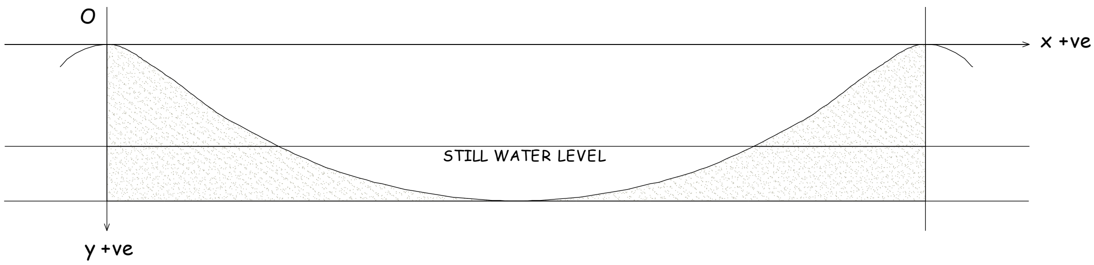


Figure 3-8

Obviously, the total volume of water must remain constant, therefore the area of the shaded section below the curve in Figure 3-8 must be exactly equal to that of the shaded section of Figure 3-7. The height of the line of orbit centres above still water level can be determined exactly by calculating and comparing the areas of each of those shaded sections.

For Figure 3-7, the area of the shaded section can be found by simply multiplying the two sides of the rectangle. The length of the horizontal side is equal to the wavelength, usually referred to as ʎ, which, in turn, is equal to 2πR. If the line of orbit centres is an unknown distance 'h' above still water level then the length of the vertical side of the rectangle is (r-h), resulting in an area of 2πR(r-h) for the rectangle.

$$Shaded\ Area_{Fig\ 3-7} = 2\pi R(r-h)$$

Calculating the area of the shaded section of Figure 3-8 is more complex, requiring the use of calculus to first determine the area between the trochoidal curve and the x-axis, and then subtracting that area from the rectangle formed by the line of troughs and the x-axis to obtain the result.

The area between the trochoidal curve and the x-axis for 0 ≤ θ ≤ 2π is calculated by integrating the curve:

$$Area_{above\ curve} = \int f(x)dy = \int (R\theta - r\sin\theta)dy$$

But, from the equation of the trochoid:

$$y = r - r\cos\theta$$

$$\frac{dy}{d\theta} = r\sin\theta$$

$$dy = r\sin\theta\, d\theta$$

Substituting in the equation above:

$$Area_{above\ curve} = \int_0^{2\pi} (R\theta - r\sin\theta)dy = \int_0^{2\pi} (R\theta - r\sin\theta)r\sin\theta\, d\theta$$

$$Area_{above\ curve} = 2\left(\pi Rr + \frac{\pi r^2}{2}\right)$$

For the rectangle formed by the line of troughs and the x-axis in Figure 3-8, the length of the horizontal side is equal to the wavelength, ʎ, which, in turn, is equal to 2πR. The length of the vertical side is 2r, resulting in an area for the rectangle of 4πRr.

$$Area_{rectangle} = 4\pi Rr$$

Therefore, by substitution, the area of the shaded section in Figure 3-8 is given by:

$$Shaded\ Area_{Fig\ 3-8} = 4\pi Rr - 2\left(\pi Rr + \frac{\pi r^2}{2}\right) = 2\pi Rr - r^2$$

But, the shaded areas of Figures 3-7 and 3-8 are exactly equal, therefore:

$$2\pi R(r - h) = 2\pi Rr - r^2$$

$$h = \frac{r^2}{2R}$$

That is, the line of orbit centres is always a distance of $r^2/2R$ above still water level.

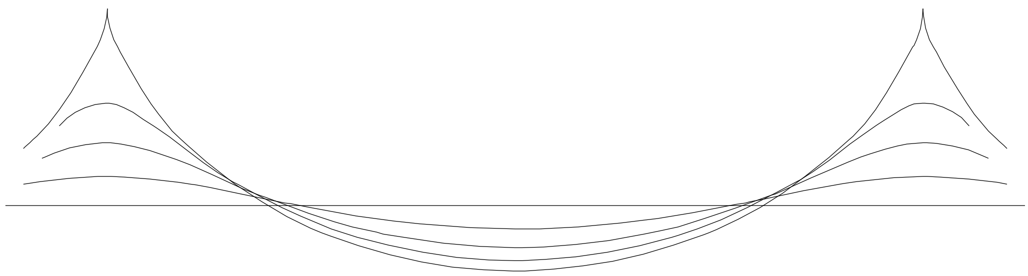


Figure 3-9

The trochoidal waves shown in Figure 3-9 are positioned relative to still water level and have the same dimensions of the trochoidal curves in Figure 3-2 which were positioned, instead, about their common line of orbit centres.

The Slope of the Wave Surface

The point P on the rolling line is, from the definition of the trochoid, the instantaneous centre of the curve at S. Consequently, the radial arm PS is normal to the curve at that point and the slope of the curve at S is perpendicular to the line PS.

In Figure 3-10 a line has been constructed from the point P, on the rolling line, to the corresponding point S, on the trochoid, and another line has been constructed from S perpendicular to the radial arm of the rolling wheel, PC.

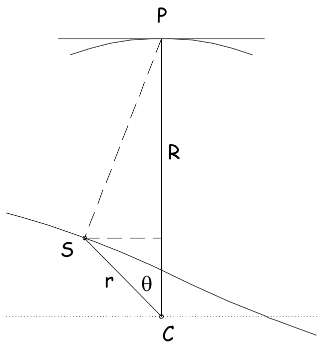


Figure 3-10

By deduction, the slope of the trochoid at S is equal to the angle CPS which can be evaluated from the constructed triangle:

$$\tan \widehat{CPS} = \frac{rsin\theta}{R - rcos\theta}$$

If the angle CPS is denoted by φ, the slope of the wave surface at S is given by:

$$\varphi = \tan^{-1} \frac{rsin\theta}{R - rcos\theta}$$

From the triangle CPS it can be seen that as S revolves anti-clockwise around C, the angle CPS, or φ, reaches a maximum value at the points on the curve where the radial arm, r, is perpendicular to the line PS. At those points the value of φ is given by:

$$\sin \varphi = \frac{r}{R}$$

In other words, the maximum slope of a trochoidal wave surface is:

$$\varphi_{max} = \sin^{-1} \frac{r}{R}$$

Using the same logic, the points of maximum slope are located on the wave surface at the points where θ, the angle of rotation of the radial arm, r, is given by:

$$\theta = \cos^{-1} \frac{r}{R}$$

Virtual Gravity and Virtual Upright

The water particle S, located on the wave surface and revolving in a circular orbit about C with a constant angular velocity, is subject to both the force of gravity, vertically downward, and centrifugal force, along the radial arm CS, outward from the centre. The centrifugal force is caused by the rotation of the particle about C and is similar to the force acting on water in a bucket as the bucket is whirled in a vertical circle.

Figure 3-11 shows the forces and their resultant which, because the wave is a free surface in dynamic equilibrium, must always be normal to the wave surface.

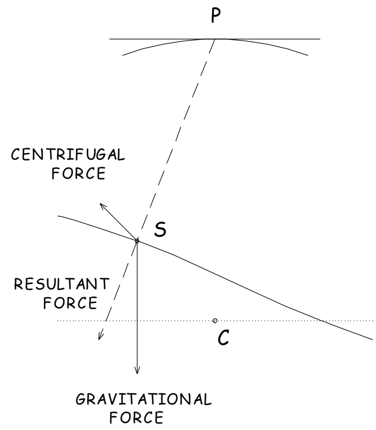


Figure 3-11

Previously, when determining the slope of the wave surface at S, it was shown that the line PS is also always normal to the wave surface. Therefore, the triangle CPS in Figure 3-10 can be used as a triangle of forces. The gravitational force acting vertically downward at S can be represented by the line PC, the centrifugal force outward from C by the line CS, and the resultant force normal to the wave surface by the line PS. The forces acting on the water particle at S and the resultant of those forces are therefore proportional to the lengths of the sides of the triangle CPS, resulting in the equations:

$$\frac{mg}{PC} = \frac{mr\omega^2}{CS} = \frac{mg_v}{PS}$$

where ω=dθ/dt, the angular velocity about C of the water particle at S, and mg_v is the resultant force or 'virtual gravity' at S.

If PS, normal to the wave surface, is denoted by n, by substitution the virtual gravity can be expressed as:

$$mg_v = \frac{n}{R}mg$$

Virtual gravity therefore changes in magnitude and direction along the wave surface in direct proportion to the value of n, becoming a maximum in the troughs, where n=(R+r), and a minimum in the crests, where n=(R-r), but always remains normal to the wave surface, creating a 'virtual upright' at each point.

The quantity mg_v is termed virtual gravity because it is the resultant force acting on the water particles on the wave surface, instead of the real gravity mg which is the only external force acting on the water particles in still water. As a consequence of the apparent changes in gravitational force along the wave, the virtual weight of a boat floating on the surface of a wave therefore varies from a maximum in the troughs to a minimum in the crests. The effects of variations in the magnitude of the virtual gravity along a wave surface are noticeable. A high-performance dinghy planing downwind, for example, can feel light and lively as it is lifted on a crest but, moments later, due to the increase in the virtual gravity, can be wallowing in a trough, feeling heavy and sluggish as if being pressed into the water surface.

Divergence of the virtual upright from the true vertical is a major contributor to sea-sickness, but not only because of the obvious effects on the human body's internal balance mechanism. At sea, on an ocean swell, the constantly changing direction of the virtual upright can also create an impression that the approaching waves are much higher than actual, adding to the anxieties of a novice crew.

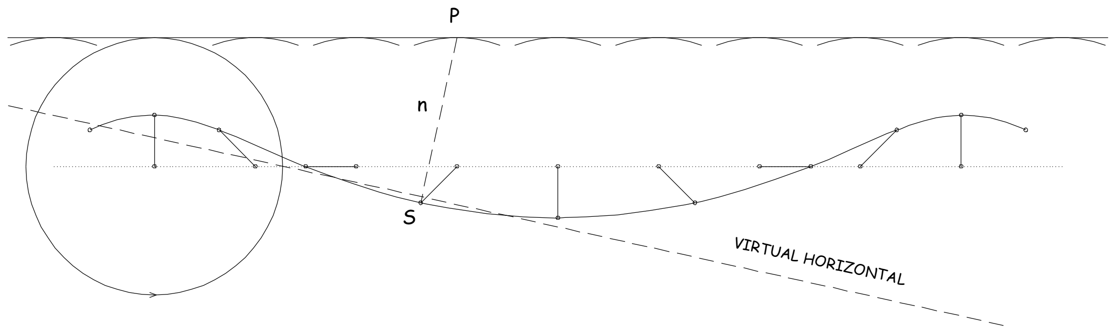


Figure 3-12

Consider, for simplicity, a swimmer treading water at S in Figure 3-12. The swimmer, whose head is practically at sea level, is being subjected to the same forces as the water particles within the wave. As shown previously, the resultant of those forces on the water surface, the virtual gravity, acts along the line PS, the virtual upright, causing the swimmer's internal balance mechanisms to react and temporarily create a corresponding 'virtual horizontal', inclined to the true horizontal as shown. If Figure 3-12 is rotated anti-clockwise so that the virtual horizontal becomes a true horizontal to give the swimmer's perspective at S, the crest to the right will appear to have increased in height, while the crest to the left will appear to have decreased in height. As the wave progresses the inclination of the virtual horizontal at S varies, giving the swimmer a constantly changing perception of the wave conditions.

This phenomenon, observed from a small vessel in an ocean swell, can create an impression that the approaching crests are abnormally high and threatening, but nearer the vessel the waves seemingly subside, passing harmlessly beneath the hull to emerge as much smaller crests.

The Velocity of the Wave

The velocity of a trochoidal wave is exactly equal to that of the rolling wheel which was initially used to create the trochoidal curve. Over a length of time, t, the rolling wheel advances a distance Rθ, resulting in a velocity given by the equation:

$$v_{rolling\ \ wheel} = \frac{R\theta}{t}$$

The term θ/t, or more precisely dθ/dt, is the angular velocity of the water particle at S, usually expressed as ω, resulting in an alternative equation for the velocity of the wave:

$$v_{wave} = R\omega$$

Previously, when investigating the magnitude and direction of the resultant force on the water particle at S, it was determined that the forces acting on the particle are proportional to the lengths of the sides of the triangle CPS in Figure 3-10, resulting in the equations:

$$\frac{mg}{PC} = \frac{mr\omega^2}{CS} = \frac{mg_v}{PS}$$

By substitution:

$$\frac{mg}{R} = \frac{mr\omega^2}{r} = \frac{mg_v}{n}$$

From which can be obtained an alternative value for ω:

$$\frac{g}{R} = \omega^2$$

$$\omega = \sqrt{\frac{g}{R}}$$

Therefore, the velocity of the wave can now be expressed as:

$$v_{wave} = \sqrt{gR}$$

The distance travelled by the rolling wheel through one complete rotation is equal to the wavelength of the trochoid, λ, therefore:

$$\lambda = 2\pi R$$

Substituting for R, a usable formula for the velocity of the wave is finally obtained:

$$v_{wave} = \sqrt{\frac{g\lambda}{2\pi}}$$

From the formula it becomes apparent that the velocity of a trochoidal wave is totally dependent on the wavelength and is directly proportional to the square root of that length. Observations of the stern waves in the wake of a vessel clearly indicate that wavelengths and wave velocities are directly related. The stern waves, which always keep pace with the vessel, have short wavelengths when the vessel is moving slowly but space themselves further apart as the speed of the vessel increases. Experiments have confirmed that the velocities of water waves of more than about 15mm in length are in fact directly proportional to the square root of their wavelengths, as determined by the above formula. For waves shorter than 15mm, which are the slowest of all water waves, the attractive forces between the water molecules become dominant and the trochoid cannot be used to represent the wave formation. As a matter of interest, those same cohesive forces between the water particles also prevent wind-generated waves from being formed on the water surface until the wind speed reaches approximately three knots.

Using the above formula, for a wavelength expressed in metres, the velocity of the wave is equal to $2.43\sqrt{\lambda}$ knots. The following table shows the relationship between wavelength and wave velocity for a selection of trochoidal waves:

Wavelength, λ (metres)	Velocity (metres/second)	Velocity (knots)
0.1	0.4	0.8
1	1.2	2.4
5	2.8	5.4
10	3.9	7.7
25	6.2	12
50	8.8	17
100	12.5	24

In contrast with waves on the water surface, most of the commonly known wave types, such as sound waves or radio waves, have velocities which are practically constant regardless of the wavelengths. Sounds produced by the various instruments of an orchestra, for example, may have many different wavelengths but all reach the listener's ear at the same time. The velocity of sound, instead of being determined by the wavelength, is more dependent on the properties of the medium through which the sound waves are passing. Nevertheless, photographs of ripples on a water surface are often used in textbooks describing general wave motion to demonstrate properties of other wave types, such as sound waves. For many of those properties the analogy is quite close, hence the reason for the term 'wave' in describing sound and similar types of energy transmission in the first place, but the unique relationship between the velocity and the wavelength of surface waves is usually overlooked.

Waves on the water surface do not all have the same velocity but, instead, each wave has a velocity directly dependent on its individual wavelength. If it is presumed that the energy contained within an advancing wave is likely to be related to the wave's velocity, then the velocities of waves on the surface of the water become of particular interest in the theory of hull design since it is the hull, moving forward, which unavoidably creates the surface waves within the visible wake and, in so doing, becomes the sole source of energy of those waves.

The Energy of the Wave

Although the wave profile advances at a velocity dependent on the wavelength, the actual water particles within the wave are not advancing but are revolving in circular orbits about fixed centres, as shown previously in Figure 3-4. The individual water particles have kinetic energy, due to their motion, and potential energy, due to their vertical displacement from their initial equilibrium position in still water. The energy of an advancing trochoidal wave or, from a different perspective, the energy required to create a wave, is the sum total of the kinetic and potential energies of the individual water particles within the wave, to a depth where the water particles remain undisturbed by the wave motion. By analysing the movement of the water particles it can be shown that for an advancing trochoidal wave as a whole, the total kinetic energy is equal to the total potential energy or, alternatively, the energy of an advancing wave is half kinetic and half potential.

Calculation of the kinetic and potential energies of an individual water particle requires, among other things, that the mass of the particle be known. As well, to calculate the total energy of an advancing wave, the number of water particles within the wave must be determined. Development of the *Trochoidal Wave Theory* has, to this point, been limited to consideration of water particles on the water surface only, the particles being infinitesimal with an undefined mass. Proceeding beyond that basic assumption to calculate the energy of a wave necessitates introducing a concept of quantifiable elements of water rather than individual water particles, each element representing, say, a collection of particles that, as a group, behave almost as one and have real dimensions, including mass.

Figure 3-13 is an exaggerated representation of a thin surface layer that has been given a nominal thickness to allow an analysis of the elements within it. The *Trochoidal Wave Theory* suggests that for water particles below the wave surface sub-trochoids exist. Consequently, the 'top' and 'bottom' of the thin surface layer can be defined by marginally different trochoids, resulting in variations in the thickness of the surface layer as shown.

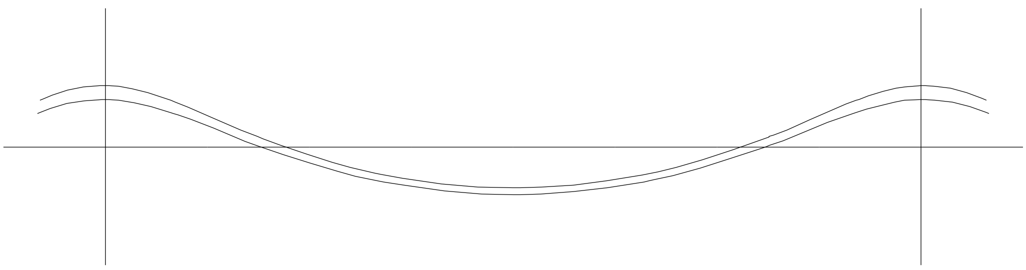


Figure 3-13

Assuming that the water is incompressible, to maintain a continuity of flow within the surface layer as the water particles shuffle backwards and forwards with the advancing wave motion, the quantity of water passing any fixed point along the stream must be constant. Logically, for a given breadth of wave, that quantity of water is a product of the thickness and the velocity of the stream at that point.

If the thickness of the surface layer is small, then the velocity of the stream can be assumed to be equal to that of a water particle on the wave surface. The stream velocity of a water particle at any point on the wave surface, depicted previously by S in Figure 3-10, is a resultant of the particle's component velocities in the directions of the x and y axes. This means that by using Pythagoras' Theorem the velocity of the particle along the water surface can be expressed as:

$$v_{stream}^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2$$

From the equation of the trochoid:

$$x = R\theta - r\sin\theta$$

$$y = r - r\sin\theta$$

therefore:

$$\frac{dx}{d\theta} = R - r cos\theta$$

$$\frac{dy}{d\theta} = r - r cos\theta$$

But:

$$\frac{dx}{dt} = \frac{dx}{d\theta}\frac{d\theta}{dt} \quad \& \quad \frac{dy}{dt} = \frac{dy}{d\theta}\frac{d\theta}{dt}$$

Therefore, the velocity of the particle along the water surface can now be expressed as:

$$v_{stream}^2 = (R - r cos\theta)^2 \left(\frac{d\theta}{dt}\right)^2 + (r\sin\theta)^2 \left(\frac{d\theta}{dt}\right)^2$$

$$v_{stream}^2 = (R^2 - 2R\cos\theta + r^2)\omega^2$$

From Figure 3-10 it can be determined that the line PS, normal to the curve at S and subsequently given the value n, has a length equal to $(R^2 - 2Rr\cos\theta + r^2)^{1/2}$ so that by substitution:

$$v_{stream} = n\omega$$

If the thickness of the thin surface layer is denoted by the variable δ, for a given breadth of wave the quantity of water passing any fixed point along the stream within the surface layer becomes nωδ, the product of the thickness and the velocity of the stream at that point.

The quantity of water passing any fixed point along the stream is constant, therefore, since ω is uniform, the product nδ must also be constant, indicating that the stream thickness varies as depicted in Figure 3-13, being a minimum in the troughs and a maximum in the crests.

$$n\delta = constant$$

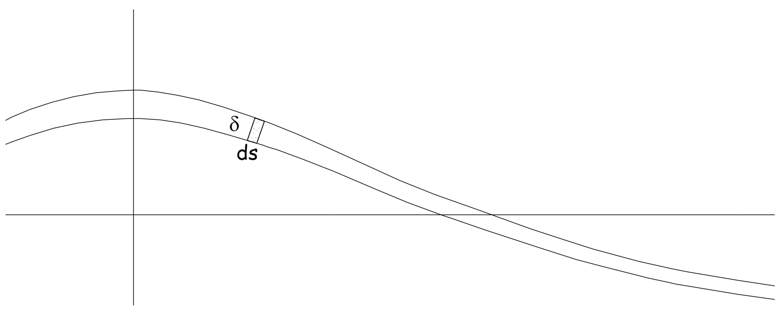


Figure 3-14

Figure 3-14 depicts a short segment of the surface layer at S, an element of water that has a depth denoted by δ, the thickness of the stream, and a length denoted by ds.

By inspection, the velocity of the element along the stream is ds/dt, but since the velocity of the stream is nω, then:

$$\frac{ds}{dt} = n\omega$$

$$\omega = \frac{d\theta}{dt}$$

therefore:

$$ds = nd\theta$$

and the area of the element can be expressed as:

$$A_{element} = \delta n d\theta$$

Consequently, the area of one wavelength of the trochoidal strip representing the surface layer, as depicted in Figure 3-13, is:

$$A_{layer} = \int_0^{2\pi} \delta n \, d\theta$$

The product, δn, is a constant, therefore:

$$A_{layer} = \delta n 2\pi$$

The trochoidal strip represents a cross-section of the surface layer of an actual wave. If the density of the water is denoted by ρ, then the mass of one wavelength of the trochoidal strip can be expressed as:

$$m_{layer} = \rho\delta n 2\pi \; per\; unit\; breadth\; of\; wave$$

Knowing the mass of the trochoidal strip it is now possible to calculate the kinetic and potential energies of the surface layer, using the general formulae $KE=\frac{1}{2}mv^2$ and PE=mgh, where v is the velocity of the surface layer and h is the height of the surface layer above its position in still water.

As the wave advances, the water particles within the surface layer revolve in circular orbits about fixed centres as depicted in Figure 3-4. Together, the rotation of the water particles defines the oscillatory motion of the surface layer which, as a consequence, has the same velocity, rω. Therefore the kinetic energy of the trochoidal strip becomes:

$$KE_{layer} = \frac{1}{2}(\rho\delta n 2\pi)(r\omega)^2 \; per\; unit\; breadth\; of\; wave$$

But previously it was ascertained that:

$$\omega = \sqrt{\frac{g}{R}}$$

Therefore the kinetic energy of the trochoidal strip can be expressed as:

$$KE_{layer} = \frac{\rho\delta n\pi r^2 g}{R} \text{ per unit breadth of wave}$$

The centre of gravity of the thin trochoidal strip representing the surface layer is, by inspection, located on the line of orbit centres about which the water particles are revolving. Previously, the line of orbit centres was determined to be at a height of $r^2/2R$ above still water level and so the potential energy of the trochoidal strip becomes:

$$PE_{layer} = (\rho\delta n 2\pi)(g)\left(\frac{r^2}{2R}\right) \text{ per unit breadth of wave}$$

Therefore the potential energy of the trochoidal strip can be expressed as:

$$PE_{layer} = \frac{\rho\delta n\pi g r^2}{R} \text{ per unit breadth of wave}$$

By comparing the results, the potential energy of the trochoidal strip is equal to the kinetic energy and the total energy for one wavelength of the trochoidal strip is equal to:

$$Energy_{layer} = \frac{2\rho\delta n\pi g r^2}{R} \text{ per unit breadth of wave}$$

The equation, containing the variables δ and n, is difficult to evaluate and needs to be rearranged so that the energy of the trochoidal strip, and ultimately the energy of the entire wave, can be expressed in practical terms. For that purpose Figure 3-15 shows a more detailed view of the segment of the surface layer referred to previously.

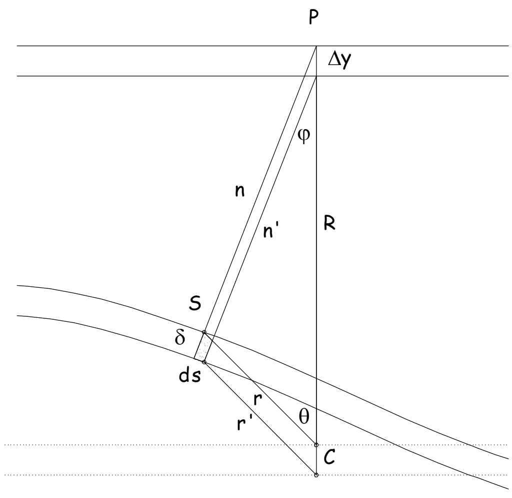


Figure 3-15

In Figure 3-15 the 'top' and the 'bottom' of the thin surface layer are represented by trochoidal curves, each with its own rolling line and line of orbit centres. Note that r' is parallel to r and, if the element of water within the surface layer bounded by n and n' is extremely small then, as $\delta \rightarrow 0$ and $ds \rightarrow 0$, n' can be considered to be parallel to n. Using that assumption it can be deduced that:

$$n + \delta = n^{'} + \Delta y \cos \varphi$$

therefore:

$$\delta = n^{'} + \Delta y \, cos \, \varphi - n$$

resulting in:

$$\delta = \Delta n + \Delta y \, cos \, \varphi \;\; where \; \Delta n = (n^{'} - n)$$

The product nδ can now be expressed as:

$$n\delta = n\Delta n + n\Delta y \, cos \, \varphi$$

As shown previously, from the triangle CPS:

$$n^2 = R^2 - 2Rr \cos \theta + r^2$$

and since, for the trochoid, R is constant and, for the instant depicted in Figure 3-15, θ is constant, by differentiating the equation with respect to r:

$$2n\frac{dn}{dr} = -2R \cos \theta + 2r$$

or, alternatively:

$$n\Delta n = r\Delta \mathrm{r} - R \cos \theta \Delta r$$

Again, from the triangle CPS:

$$n\cos\varphi = R - r \cos \theta$$

Bringing the equations together:

$$n\delta = n\Delta n + n\Delta y \, cos \, \varphi$$

therefore:

$$n\delta = (r\Delta \mathrm{r} - R \cos \theta \Delta \mathrm{r}) + (R - r \cos \theta)\Delta y$$

resulting in:

$$n\delta = r\Delta r + \mathrm{R}\Delta y - \cos \theta (R\Delta r + r\Delta y)$$

Previously it was shown that nδ is constant and for that to be possible for all values of θ then (RΔr + rΔy) must equal zero, therefore:

$$R\Delta r = -r\Delta y$$

resulting in:

$$n\delta = r\Delta r + \mathrm{R}\Delta y \; where \; \Delta y = -\frac{R\Delta r}{r}$$

and, by substitution:

$$n\delta = \Delta r\left(\frac{r^2 - R^2}{r}\right)$$

Using this more practical value of nδ, the equation used to determine the energy of one wavelength of the trochoidal strip on the wave surface can now be expressed as:

$$Energy_{layer} = \frac{2\pi\rho g}{R}(r^3 - rR^2)\Delta r \; per\; unit\; breadth\; of\; wave$$

The total energy of one wavelength of a trochoidal wave can now be obtained by the summation of the energies of all of the trochoidal strips below the surface, to a depth where r=0. If r_0 is the value of r at the surface, by integrating the energy equation with respect to r then:

$$Energy_{wave} = \frac{2\pi\rho g}{R}\int_0^{r_0}(r^3 - rR^2)\,dr \; per\; unit\; breadth\; of\; wave$$

therefore:

$$Energy_{wave} = \frac{2\pi\rho g}{R}\left(\frac{r^4}{4} - \frac{r^2R^2}{2}\right)_0^{r_0} \; per\; unit\; breadth\; of\; wave$$

$$= \rho g\frac{2\pi}{R}\left(\frac{{r_0}^4}{4} - \frac{{r_0}^2R^2}{2}\right)$$

$$= \frac{1}{2}\rho g\frac{2\pi R^2}{R}\left(\frac{{r_0}^4}{2R^2} - {r_0}^2\right)$$

$$= \frac{1}{2}\rho g\lambda {r_0}^2\left(\frac{{r_0}^2}{2R^2} - 1\right)$$

But, since r<R and the energy of a wave is positive, the energy of one wavelength of the trochoidal wave can be expressed as:

$$Energy_{wave} = \frac{1}{2}\rho g\lambda r^2\left(1 - \frac{r^2}{2R^2}\right) \; per\; unit\; breadth\; of\; wave$$

The Velocity of a Wavetrain

Ocean waves, generated by stormy weather at sea and sent crashing onto far-off coastlines, often days later, leave little doubt that surface waves can transport enormous amounts of energy over long distances. But, Gerstner's wave theory suggests that the waves have only an appearance of power, their perceived energy belonging, instead, to individual water particles rotating in stationary orbits on and beneath the surface. Questions immediately arise as to how, then, is the energy actually transported from one location to another and at what velocity, if not conveyed by the waves themselves. Clues are to be found, not by studying the incessant procession of ocean waves at sea but by carefully observing the mere ripples that occur when a stone is thrown into a calm pond.

After a stone strikes a water surface, the disturbed water resolves itself into a circular pattern of waves which moves out equally in all directions. Surprisingly, as the ring of waves expands, the waves travel outward in a group, or wavetrain, leaving behind a quiescent centre. Within the group, individual waves can be seen to emerge from the calm water at the trailing edge, travel outward faster than the group is advancing as a whole and disappear into the calm water ahead as they reach the leading edge. When first viewed, the outcome is unexpected and appears to be an optical illusion but the astute observer will recognise that a similar situation also occurs as the bow wave of a vessel advances into calm water. In each case the viewer is witnessing a phenomenon that is fundamental to the study of wave motion generally, a group velocity that is lower than the velocity of the individual waves within the group. From observation alone it is immediately apparent that the velocity of the group, not that of the individual waves, is the rate at which the energy advances.

Gerstner, when he formulated the *Trochoidal Wave Theory*, first published in 1804, may have been unaware of the concept of a distinct group velocity, formally proposed for general wave motion more than three decades later in 1839, by an Irish mathematician, William Rowan Hamilton. Since that period, the initial disturbance of a liquid surface and the subsequent wave motion caused by an impact such as that of the stone being thrown into a calm pond, have been the focus of on-going analysis by physicists and mathematicians attempting to unravel the secrets of all types of wave motion. As might be expected, an exact explanation of what actually occurs near the point of impact is elusive and the investigations are mathematically complex. But, looking beyond the initial disturbance to a point where the water has resolved itself into a circular pattern of waves moving outward with a constant velocity does allow a more simplistic approach and, by applying the relatively simple concepts of Gerstner's *Trochoidal Wave Theory*, the group velocity of waves on the water surface can be determined without difficulty.

Prior to contact with the water, the stone in the example above has kinetic energy due to its velocity. On impact with the water surface, a portion of that kinetic energy is imparted to the water particles with which the stone collides, reducing the velocity of the stone and setting the water particles in motion. The extent of the initial disturbance and the amount of kinetic energy transferred depends not only on the mass and the velocity of the stone but also its shape. In turn, the affected water particles collide with adjacent particles, causing the impulse of the initial collision to be relayed outward from the disturbance by a process that is not fully understood but, from observation, necessitates the creation of a series of waves. Within those waves the water particles rotate in circular orbits but, most significantly, each particle rotates about a fixed centre that is located above the particle's 'at rest' position in calm water. In other words, the kinetic energy imparted to the water by the stone creates a wave motion within which the individual water particles not only gain kinetic energy by being put into circular motion, but also gain potential energy by being raised above their initial equilibrium position in still water.

As the ring of waves expands outward from the disturbance, the kinetic energy of each rotating particle within the waves is continually being renewed by collisions from the particles behind. At the trailing edge of the wave group the water particles, having expended all of their energy in activating the particles ahead, return to their 'at rest' position, creating the observed quiescent centre of the disturbance. Similarly, as the group of waves advances into calm water, the water particles at the leading edge of the group are obliged to provide sufficient energy to not only put undisturbed particles in motion but to also lift those particles above their 'at rest' positions. More precisely, the kinetic energy that the particles at the leading edge of the group have because of their motion is required to provide both the kinetic energy and the potential energy of the adjacent particles in the undisturbed water ahead. Kinetic energy is transferred as the particles collide but clearly, the kinetic energy of a lone particle within a wave is insufficient to provide an adjacent water particle with an equal amount of kinetic energy plus the energy needed to raise that particle above its 'at rest' position in calm water. Gerstner's wave theory reasons that the combined potential energy of all of the activated water particles within one complete wavelength is equal to their combined kinetic energy, leading to the logical conclusion that the combined kinetic energy of all of the particles within two complete wavelengths is necessary to provide the energy required to advance a wavetrain by one wavelength into undisturbed water. By simple deduction, the velocity of a wavetrain on the surface of the water is, therefore, half that of the individual waves within the group.

$$v_{wavetrain} = \frac{v_{wave}}{2}$$

A Summary of the Trochoidal Wave Theory

Gerstner's *Trochoidal Wave Theory*, developed at the beginning of the 19th century, assumes a perfect, non-viscous, incompressible, elastic fluid of infinite depth and extent. Although water can never be that perfect fluid, observations of the behaviour of ocean waves in deep water fit so well with the *Trochoidal Wave Theory* that the theory is now generally accepted, at least for elementary applications.

The mathematical equations that define the principal elements of the trochoidal wave are:

$$x = R\theta - r\sin\theta$$

$$y = r - r\cos\theta$$

$$h = \frac{r^2}{2R}$$

$$v_{wave} = \sqrt{\frac{g\lambda}{2\pi}}$$

$$KE_{wave} = PE_{wave}$$

$$Energy_{wave} = \frac{1}{2}\rho g\lambda r^2\left(1 - \frac{r^2}{2R^2}\right) \text{ per unit breadth of wave}$$

$$v_{wavetrain} = \frac{v_{wave}}{2}$$

A WAVELINE THEORY

The phenomenon of wave motion occurs in almost every branch of modern physics and the investigation of the properties and behaviour of waves has long been of great importance to the scientific community. A comprehensive wave theory of light, for example, was proposed by Christiaan Huygens as early as 1678, while the use of the analogy of water waves to describe the propagation of sound as a disturbance of the air is believed to have originated during the ancient Greek era. Today, science is familiar with many types of waves but waves on a water surface are, by far, the most easily observed and it is not surprising that water waves were the initial focus of mathematical investigations to unravel the mysteries of wave motion.

As an incentive to extend the existing knowledge of waves, in 1813 the French Academy of Sciences offered a prize for the mathematical solution to the propagation of waves caused by a sudden disturbance on the surface of a liquid, a single splash. The prize was eventually earned by a twenty-five year old French mathematician, Augustin-Louis Cauchy and his exhaustive work was later extended by one of the competition's judges, Siméon Poisson. Although the Cauchy-Poisson analysis is now accepted by mathematicians as a milestone in the development of the mathematical theory of initial-value problems, originally there had been a negative reaction to their achievement. For Cauchy's and Poisson's compatriots, the mathematics was not only too demanding but the calculated results were confusing, appearing contrary to intuition. In Britain, few mathematicians understood or even bothered to study the works of their mainland counterparts, considering their accomplishments to be little more than uninteresting and useless forays into the world of pure mathematics. British mathematicians, living within an island kingdom whose Royal Navy 'ruled the waves', were more concerned with the less obscure but, in their judgement, the more practical and more worthy aspects of hydrodynamics such as the construction of sea walls and harbours, the determination of tidal flows, and the design of ships. Unpredictably, however, throughout the 19th century, via an uncoordinated, almost haphazard process of experimental and mathematical investigation, from Britain were to emerge significant advances, not only in ship design theory but also in the theory of wave motion generally.

In 1833, an assessment of the state of hydrodynamics within Britain was commissioned by the British Association for the Advancement of Science. The subsequent report of that review concluded with a pessimistic comment on the domestic scientific community's general lack of understanding of fluid resistance, a crucial factor in ship design that needed to be resolved if Britain's superiority at sea was to be preserved during the transition from sail to steam. To confirm the ineffectiveness of existing ship design theory, the report cited the example of a seemingly mundane incident involving a boat being towed by a horse along a canal near Glasgow, Scotland. Members of the British Association, whose ranks included eminent mathematicians and physicists, were informed of how on one occasion a horse had bolted, dragging a canal boat at high speed. To the amazement of the owner, William Houston, the boat had jumped over its bow wave and continued moving along the canal through comparatively smooth water with reduced resistance. Astern, the wake flattened and the usual surge, which previously had been damaging the banks of the canal at lower speeds, had almost disappeared. This particular incident had so impressed Houston that a system of 'swiftboats' to carry passengers between local towns at speeds of about 10 knots had since been introduced, halving the previous travel times. The largest of the new boats exceeded 20 metres in length and were each capable of holding as many as ninety passengers. Today, we might nonchalantly describe the performance of the canal boat as merely 'planing', having sufficient reserve power to climb up and over its bow wave, allowing the boat to surf the wave with considerably less effort. But, in 1833, the concept of planing was unfamiliar and, in the narrow and shallow canal, there were also other forces at work. Existing hydrodynamic theory was unable to provide a satisfactory mathematical explanation of the phenomenon but investigations following that incident did lead to another accidental discovery, this time by a local engineer.

John Scott Russell was born in a village near Glasgow in 1808, the son of a minister of religion and destined to follow in his father's footsteps. But, the Industrial Revolution had spread rapidly and Britain was in the midst of emerging from its previous agrarian, handicraft economy to one dominated by industry. British way of life was changing and Russell, no doubt influenced by the sights and sounds of the activities at nearby coal mines and having an aptitude for practical science, abandoned his planned career in the church and studied engineering instead, graduating from spells at St Andrews, Edinburgh and Glasgow universities in 1825. At age 24, while working as a lecturer in science at Edinburgh University, Russell was provisionally appointed as a professor of natural philosophy, the position having become vacant by the death of his predecessor. In that role, following the incident of the horse that bolted, Russell was consulted by the Union Canal Company regarding the possibility of steam-navigation on the Edinburgh and Glasgow Union Canal. The ensuing research would mark the beginning of a refreshing re-evaluation of ship design theory in Britain and a turning point in Russell's career.

Within the confines of a canal the performance and the wake of a boat vary from when in open water, primarily caused by the relatively shallow depth of a canal and the proximity of the canal walls. Russell, presumably with little practical experience in either boat design or handling, prudently decided that the feasibility of introducing high-speed navigation to the canal system could not be properly determined without experiment. Accordingly, during the summers of 1834 and 1835 a length of canal was placed at Russell's disposal and with the help of two assistants and a dozen hired hands, four different canal boats were built and their performances measured. Initially, horses provided the towing force but for the second summer were replaced by the falling weight method, pulling the full-size vessels over a range of speeds to a maximum of about 15 knots. During that first summer Russell made an observation that is best described in his own words, extracted from the Report of the fourteenth meeting of the British Association for the Advancement of Science, York, September 1844:

> *"I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped - not so the mass of water in the channel which it had put in motion; it accumulated round the prow of the vessel in a state of violent agitation, then suddenly leaving it behind, rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed. I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour, preserving its original figure some thirty feet long and a foot to a foot and a half in height. Its height gradually diminished, and after a chase of one or two miles I lost it in the windings of the channel. Such, in the month of August 1834, was my first chance interview with that singular and beautiful phenomenon which I have called the Wave of Translation".*

Russell described the solitary wave as a 'wave of translation', drawing attention to the fact that he believed the wave which he had observed involved an actual mass transport of water from one end of the canal to the other, distinguishing the solitary wave from the usual oscillatory waves where no real mass transport occurs. The velocities of the solitary waves in the canal were subsequently measured and found to be dependent on the height of the wave and the depth and width of the canal.

For the canal boats, Russell observed that the shape of the water surface around the hull varied according to the speed of the boat and that water was raised at the bow, becoming higher as the boat's speed was increased. Logic suggested that the measured rise in the boat's resistance at those higher speeds was caused by the change in the longitudinal inclination of the hull necessary to push the bow wave along, effectively altering the hull's immersed frontal area. Surprisingly, however, at a critical speed dependent on the depth of the canal, measurements consistently revealed that the boat's resistance suddenly lessened. Applying his newly found knowledge of the solitary wave, Russell reasoned that when the canal boat reached the critical speed, the velocity of the bow wave was then equal to the natural velocity of the solitary wave and, as a consequence, the bow wave no longer needed the push of the hull for its advancement, thereby reducing the resistance of the boat. By maintaining the boat's propulsion, the boat was then able to overtake the bow wave and proceed on the face of the wave at the wave's natural velocity. Unwittingly, Russell had outlined, for the first time, an explanation of the progression of a vessel passing though the critical barrier which today we loosely refer to as 'hull speed'.

Convinced that he had witnessed something unique in the solitary wave, Russell resolved to personally investigate the phenomenon more closely and, to replicate the experience, built a water tank at his home in Edinburgh. Unlike many previous experimentalists, who had used tanks to research the behaviour of ships, Russell, over the next three years, examined more thoroughly the properties of the solitary wave. Injecting additional water at one end of the tank by the use of a lock system as shown in Figure 4-1, Russell was able to produce a disturbance which consistently evolved into the perfectly stable solitary wave as it propagated along the channel. The wave seemed sinusoidal in shape and Russell managed to derive a formula for its velocity, taking into account the height of the wave and the depth of water in the tank. During his experiments Russell also examined the behaviour of other wave types, 'corpuscular waves', his term for compressive sound waves propagating through water, small capillary waves and the normal oscillatory waves, noting their particular peculiarity of advancing, as a group, more slowly than the individual waves within the group. All along, Russell remained convinced that the solitary wave, his 'Great Wave of Translation' which he regarded as almost a self-sufficient entity, was of fundamental importance.

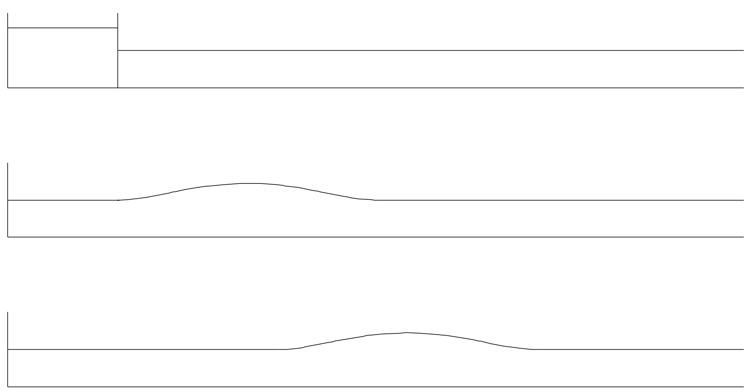


Figure 4-1

By the time of Russell's experimental work, the age of steam-navigation at sea had already arrived, the first all-steam crossing of the Atlantic having been made in 1827, by the British-built paddle steamer *Curacao.* From a scientific viewpoint steam-navigation was, in many ways, a godsend. In complete contrast to vessels under sail, the maximum speed of a steamship could be attained in absolutely calm weather, allowing onboard observers to study the performance of the ship and to personally view the waves created in its wake. As well, steamships could be run at constant speeds, allowing, for the first time, progressive speed trials to be undertaken with the intention of obtaining a better understanding of ship design theory. From a designer's viewpoint, the efficiency of a ship's progress at higher speeds was no longer peripheral. Under sail the wind had been free and speeds variable, but under the power of steam speeds were consistently higher and the necessity to store extra coal in a poorly designed vessel not only deprived a ship of valuable cargo space, large coal bills could quickly consume any advantage gained by faster voyages. As anticipated, the introduction of the steam engine was turning the maritime world on its head and, by chance, with the emergence of iron as the material of choice for hull construction, Russell's home town, Glasgow, was a natural contender to become a major competitor in the new shipbuilding industry. Being an engineer whose passion in life was not science for its own sake but, rather, its practical application, Russell's thoughts naturally turned to the relevance of the solitary wave to steamship design.

Steamships had already established that wave resistance was proving to be an obstacle to attaining higher speeds, regardless of the power of the engines, and Russell, having become an authority on waves and resistance, offered an explanation. In the shallow depths of a canal, where solitary waves could be formed, Russell had already observed that the major obstacle to reaching higher speeds could be overcome, but for deep ocean water he reasoned that there was no alternative to the wave resistance increasing with the ship's velocity. Regarding the bow wave as the 'wave of displacement' and the major contributor to resistance, Russell suggested that for moderate speeds the forward waterlines of a ship should be hollow, preferably in the shape of a curve of versed sines to approximate the observed profile of a solitary wave. Russell argued that the ship would thereby experience a minimum of resistance, claiming that the vertical cross-sections of the waveline-shaped bow would tend to produce a wave of translation on the water's surface without undue fuss. For the afterbody, Russell had concluded from his canal experiments that the water, in trying to fill the cavity left by the ship, tended to form itself into an ordinary oscillating trochoidal wave, which he termed the 'wave of replacement'. For the shallow depths of the canal, observations had indicated that the water particles rushing into the cavity were forced to move horizontally but, for deep water, Russell reasoned that most of the replacement water particles would be expected to rise vertically from the depths. As a consequence, rather than defining the shape of the waterlines aft, Russell maintained that, instead, the vertical buttock lines of a ship in deep water should be shaped in the form of a trochoidal curve to yield a minimum of resistance.

Prior to Russell's 'waveline theory', erroneous conclusions arrived at by testing models at high speed had given rise to the popular 'cod's head and mackerel's tail' concept of sailing ship design, resulting in a maximum waterline beam at a point about one third of the waterline length from the bow. As a consequence, a typical hull had a forebody about half the length of the afterbody, resulting in the sailing ship's characteristic bluff bow with a fine run towards the stern. Russell insisted, however, that for navigation in deep water the forebody of a steamship should have a length related to that of a solitary wave travelling at the ship's design speed and that the length of the afterbody, applying František Gerstner's wave theory, should be equal to half the length of a trochoidal wave travelling at that same speed. In practice, Russell's *Waveline Theory* produced a hull shape having a forebody that was equal to one and a half times the length of the afterbody, reversing the previous trend and resulting in hulls with fine bows and full stern sections. Calculation of the lengths of the entrance and run of a steamship according to Russell's theory was straightforward and unalterable. A ship's design speed of 10 knots, for example, meant that the trochoidal wave following the ship at the same speed would need to be 16.9 metres in length, resulting in an afterbody length of half that figure, about 8.5 metres, and a forebody length, at one and a half times the afterbody, of about 12.7 metres. Interestingly, Russell's theory placed no restrictions on the total length of the ship for a given speed, nor the beam, the draft, the shape of the mid-section nor any other dimension, potentially freeing designers of many of their traditional constraints. Depending on the ship's purpose, any length of 'middle body' could be inserted between the forebody and the afterbody to gain the required hull capacity, as depicted in Figure 4-2. As well, depending again on the intended use of the ship, the beam could vary independently, as could the hull's draft and cross-sectional shape. However, supposedly freed of many of their time-honored rules of thumb, ship designers remained cautiously hesitant about totally abandoning their long-established methods in favour of an unproven theory.

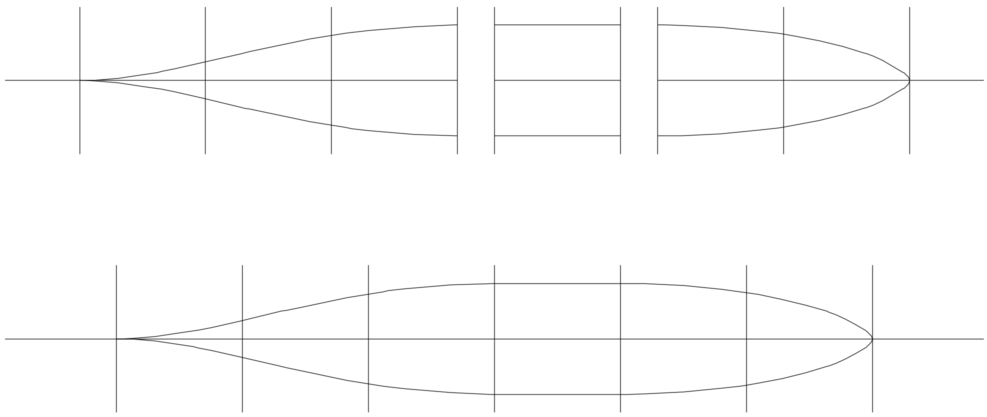


Figure 4-2

Amongst academics, Russell's experimental work on waves and his subsequent development of the *Waveline Theory* for the design of steamships received a mixed reaction. In 1837, he was awarded the gold medal of the Royal Society of Edinburgh but elsewhere Russell's efforts were criticised, no more so than by one of the nation's most eminent physicists, George Biddell Airy, the Astronomer Royal. Airy, in particular, was unimpressed by Russell's claim that he had discovered a new primary wave, asserting that the existence of the solitary wave was already known amongst mathematicians and that Russell's observations merely confirmed the complex mathematical theory of shallow-water waves derived by Joseph-Louis Lagrange in the 18th century. Although Russell was often commended for his intuition and the quality of his experimental work, academics generally were of the opinion that his mathematical interpretations suggested, at times, an ignorance of some of the elementary principles of Newtonian mechanics and subsequent advances in theoretical hydrodynamics. Despite Russell's insistence on the significance of the solitary wave, the relevance of his discovery was not immediately appreciated by mathematicians and physicists but his observations did, however, have an immediate impact, stimulating a renewed interest in the study of fluid resistance and ship design theory.

Russell, disappointed with the reception his discovery of the solitary wave had received, remained determined to succeed and in 1844 moved to London to pursue his interest in ships and the promotion of the *Waveline Theory*. Within the decade he had been appointed as a director of a shipbuilding company and became increasingly involved in ship design and construction. Amongst Russell's later achievements, perhaps none is more notable than the creation of the *Great Eastern*, a massive steamship built to the *Waveline Theory* for Isambard Kingdom Brunel, an engineer who had amassed a fortune from the construction of railways and bridges throughout Britain. Russell supervised the construction of the *Great Eastern* after collaborating with Brunel on the final design, intended to carry passengers and cargo to India and Australia without refueling en-route. At the time of her launching in 1858, after five years of assembly, the *Great Eastern* was the largest ship in the world, twice the length of any previous vessel and the biggest movable object ever built. Constructed almost entirely of iron, the *Great Eastern's* dimensions were staggering, having an overall length of about 210 metres, a beam of 25 metres and a loaded displacement of more than 30,000 tonnes. At sea, she was capable of transporting four thousand passengers in comfort at speeds to 14 knots, powered by six boilers driving two paddle wheels and a propeller. Six masts carried auxiliary sails. If successful, the *Great Eastern* would rival the last major survivors of commercial sail, the clipper ships. Unfortunately, Brunel died the year after the ship's launching and although the innovative design of the *Great Eastern* was a triumph and her construction a momentous leap forward, she proved to be a commercial failure, a setback from which Russell never fully recovered. The *Great Eastern* was withdrawn from service as a passenger ship in 1864.

Ultimately, the success, or failure, of any vessel is determined by many factors, occasionally random but usually interdependent. Russell, inexperienced as a ship designer when he first promoted his *Waveline Theory*, was almost certainly unaware in the beginning of the other improvements to ship design that the application of his theory might allow and, equally, the problems that it might cause. For experienced ship designers, the application of Russell's *Waveline Theory* to steamship design had, from the outset, the potential to provide practical advantages besides solely attempting to produce an efficient hull shape of least resistance. Increased fullness of a ship's afterbody at the waterline allowed cargo and accommodation to be concentrated further aft where there was less motion in a seaway, thereby removing unnecessary weight from the bow and improving the handling qualities of the ship. Positioning the maximum waterline beam amidships could improve stability and the elimination of the traditional bluff bow enhanced sea-going qualities. Hollow bows, the essential element of Russell's *Waveline Theory*, were not new, particularly amongst smaller sailboat designs intended for rough water, such as those of the coastal fishing fleets. But, one particular shortcoming of Russell's theory was that the very fine waterlines forward caused the bows of waveline-designed ships to plunge into the sea, making those vessels very 'wet', a failing particularly relevant to steamships, which, unlike vessels under sail, had the exceptional capability of heading directly upwind and into oncoming waves at speed. As always, in the Age of Steam the most important practical challenge for the ship designer remained unaltered, the selection of an appropriate hull form to suit the intended purpose of the vessel. For designers and builders alike, Russell's *Waveline Theory* offered advantages and disadvantages, and although designers were prepared to take onboard the advantages, whether to choose the hollow bow over a straight or a convex bow was still very much open to debate.

Robust scientific support for hull shapes other than those prescribed by Russell's *Waveline Theory* first surfaced in the 1860s in the form of a revival of the streamline theory by William John Macquorn Rankine, a professor of civil engineering and mechanics at the University of Glasgow. Born in Edinburgh in 1820, Rankine had displayed, from an early age, an exceptional aptitude for mathematics and scientific investigation. So much so, that at the age of 14 he was given, as a gift from his family, a copy of Newton's *Mathematical Principles of Natural Philosophy* in the original Latin. From age 18, after studying at Edinburgh University, Rankine pursued a career in civil engineering, maintaining a strong interest in the scientific research of almost every aspect of mechanical science, particularly that of the steam engine for which he developed and publicised a complete theory in the 1840s. By the time of his appointment as a professor in 1850, Rankine already had a working relationship with shipbuilders on the River Clyde and soon turned his attention to the theory of naval architecture, with the aim of elevating the profession into a more disciplined branch of engineering science.

Lured, no doubt, by the failure of previous attempts by mathematicians to completely solve the mystery of a ship's resistance, Rankine began his own investigations. Adapting his practical and theoretical knowledge of hydrodynamics, such as the flow of fluid through pipes, Rankine applied complex mathematics to the designs of ships being built to estimate their resistance in advance, primarily to determine the power required of their engines. Working closely with ship designers and builders, Rankine also drew attention to the fact that water particles disturbed by the passage of a ship follow neither the horizontal waterlines nor the vertical buttock lines but, instead, intermediate paths, approximately along the lines of shortest distance. In layman's terms, the paths of the water particles, the streamlines, were described as roughly being the lines of evenly spaced timber battens bent to the shape of the underwater sections of the hull, resulting in the now customary use of diagonal or ribband lines in design drawings. Rankine emphasised that it is along the streamlines that the hull imparts thrust to the water particles, and vice versa, rather than along the horizontal waterlines or vertical buttock lines as Russell had intimated. Ultimately, from Rankine's analyses of the resistance of ships there were no major breakthroughs in ship design theory nor, for that matter, any definite conclusions regarding the optimum shape of a hull to minimise resistance but Rankine's revival of the streamline theory did raise serious doubts about the logic of Russell's waveline concept.

Hydrodynamic theory, through the application of advanced mathematics, had led to the curious notion in the mid-19th century that a deeply submerged, streamlined object moving through a perfect fluid with uniform velocity could experience zero resistance. Intuitively, the idea seemed absurd but the effortless ease with which fish move through water tended to substantiate the possibility. According to the streamline theory, the energy expended in separating the fluid at the front of a moving object could be restored by the streamlines converging at the rear, in much the same way that, say, a pumpkin seed can be propelled by having its tapered end squeezed between thumb and forefinger. Drag, it was claimed, was only caused when eddies formed in the fluid and, therefore, the design objective for submerged objects should always be to encourage the streamlines to flow smoothly. Proponents of the streamline theory reasoned that, by applying the same logic to a ship travelling across the water surface, the energy expended in creating the bow wave should be able to be restored by the force of the water closing in around the stern. Unlike for a fully submerged object, however, it was expected that for surface vessels some energy would inevitably be lost in the wave-making process, particularly by the dispersal of the bow wave, which moves outwards rather than closely following the shape of the hull. All things considered, the streamline theory generally encouraged fine lines for a ship's hull and a longer run than that determined by Russell but, unlike the *Waveline Theory*, set no definite guidelines for a ship's proportions, throwing wide open, once more, the debate regarding the most efficient hull form. For ship designers, whether advocates of Russell's *Waveline Theory*, a streamline theory or, more likely, combinations of both, the only certain method of selecting the most appropriate hull shape to suit the intended purpose of a vessel reverted, yet again, to experimental investigation.

Determined to explore the matter of a ship's resistance more closely, in 1865 a retired railway engineer living in the south of England, William Froude, began a series of private tests using models constructed from two designs, the *Raven*, hollow-bowed and based on the *Waveline Theory*, and the *Swan*, a full-bodied design of Froude's own making, based on the shapes of waterbirds. Over the next two years, two sets of models, the smaller almost a metre in length and the larger about twice that size, were towed within the harbour at Dartmouth from outriggers fitted to either side of a steam-powered launch. Throughout the tests precise measurements of the resistance of each model and the speed of the launch were constantly recorded on ingenious instruments devised by Froude, the utmost care having been taken to minimise the possibility of extraneous errors. To the surprise of the shipbuilding industry the *Swan*, except at very slow speeds, consistently experienced less resistance than the *Raven* and, as a consequence, the indisputable conclusions from Froude's research effectively relegated Russell's *Waveline Theory* to history, once and for all. A totally unexpected outcome, particularly for academics and ship designers, was the subsequent announcement by Froude that his research had failed to establish any principle by which a form of least resistance could be determined, other than that the most efficient hull shape was likely to be different for each individual ship and, for each ship, different for every speed. Instantly, the objective which had been the focus of so much theoretical and experimental investigation since the time of Newton, a unique hull shape that would experience a minimum of form resistance, seemed a hopelessly naive ideal. Froude, highlighting the positive outcome of his work, contended that his tests had, at least, successfully demonstrated that models of rational size, under the right conditions, could be relied on to truly represent the ships of which they are models, allowing the essential on-going experimentation to continue at a fraction of the usual expenditure. Acceptance of Froude's findings prompted an immediate review of ship design theory regarding form resistance and a re-evaluation of the practicality of using models for scientific investigation, ultimately steering ship designers towards the tank testing methods commonly used by naval architects today.

John Scott Russell died in 1882. Much of his early experimental work had been ignored, his outspoken criticism of the conservatism of the British scientific community had left him with enemies in high places, the construction and commercial failure of the *Great Eastern* had led to bankruptcy and the *Waveline Theory*, which had directed most of Russell's working life, was flawed. Yet, Russell is now rightfully regarded as a leading figure of his time in the advancement of ship design from being solely an art towards becoming a science. In retrospect, the *Waveline Theory* could scarcely be called a failure. Due to Russell's initiatives, over a relatively short period of time much of the guesswork had been removed from the ship design process, the *Waveline Theory* had given designers freedom in determining the shape of a hull and, in so doing, had encouraged debate, research and experimentation which inevitably resulted in significant improvements to the design of ships. Russell's experimental work and his ridicule of the scientific establishment had spurred British scientists to new heights, stimulating a revival in theoretical hydrodynamics in Britain, even if only intended to prove him wrong. If there was any consolation for Russell in later life it may have been that the mathematical existence of the solitary wave, his 'Great Wave of Translation', was finally verified in 1871, not by British mathematicians but by a French theorist, Joseph Boussinesq. Almost another century would pass before physicists in the 1960s, using digital computers to study wave propagation, would begin to realise the significance of Russell's discovery. Today, the general theory of solitary waves, or solitons, has multiple applications in science beyond hydrodynamics, including electronics and, in particular, fibre optics.

THE WAKE

William Froude was already prominent within the British shipbuilding industry when he began his model testing at Dartmouth in 1865. Two years younger than John Scott Russell, Froude had been born into a wealthy rural family at Dartington, Devon, and after studying at Oxford, where he excelled in mathematics, like Russell, had pursued a career in engineering. In 1833 Froude began his professional life, first as a surveyor setting out railway construction works and later joining Isambard Brunel's staff, in 1837, as a junior engineer on the construction of the Bristol to Exeter railway. While in Brunel's employ, Froude introduced an alternative approach to the design of skewed masonry arch bridges and developed an empirical method of setting out track transition curves, necessary to ease heavy steam locomotives and carriages through bends without toppling over or forcing the tracks apart. Impressed by Froude's theoretical and practical ability, in 1841 Brunel appointed him as manager of the railway's construction and soon came to regard Froude as not only a senior member of his staff, but also a personal friend. Unexpectedly, in 1846, aged 36 and married with children, Froude ceased working for Brunel and retired to his rural birthplace to assist his father who was suffering a chronic illness. Over the next thirteen years, until his father's death, Froude assumed the role of a country gentleman but his enthusiasm for engineering never waned. At local agricultural shows, for instance, Froude, being an engineer of note, was naturally invited to adjudicate on farm machinery being exhibited but, besides judging, Froude could not resist devising methods to improve the efficiency of the equipment he was evaluating, often manufacturing enhancements afterwards in his home workshop. Likewise, his lifelong love of boating, evidenced early in life by scribbled sketches in the margins of his school books, coupled with his investigative nature, led Froude to conduct useful experimental work with screw propellers and model boats on the local waterways in his free time. During this lengthy period of 'early retirement' Froude also became the owner of a paddle steamer at Dartmouth, a director of the Dartmouth Steam Packet Company and a member of the Dartmouth Harbour Commissioners.

In 1857, the year before the launching of the *Great Eastern,* Froude was approached by Brunel for independent advice on two matters about which he was becoming increasingly anxious, concerns associated with slipping the *Great Eastern* and, assuming that the ship could be launched, her likely behaviour in a sea. Brunel's uncharacteristic uneasiness was probably typical of any owner commissioning a new vessel, exaggerated by the fact that his massive ship, besides being a huge financial investment, was, in many respects, incomparable to any vessel ever built. Froude, after an inspection of the ship's construction site at Russell's yard, was able to allay Brunel's concerns regarding the launching but, as Brunel was surely aware, the issue of the *Great Eastern's* rolling behaviour at sea depended not only on the shape of the hull but on the weight and location of every structural component, as well as that of every major item onboard. In principle, determining the ship's centre of gravity and the distribution of weight within the ship was not difficult but the calculations were so extensive that for any ship, let alone a ship the size of the *Great Eastern,* they were not normally attempted. In the past, under sail, the excessive rolling of ships at sea had never been considered to be anything other than an occasional problem since, in any case, sails effectively acted as stabilisers in most weather conditions. Under steam, however, rolling was generally accepted as inevitable and, since nothing could be done to prevent waves at sea, most ship designers considered it pointless to pursue the matter in any detail. Froude, as meticulous as ever, disagreed. Brunel's staff were subsequently given the onerous task of carrying out the tedious calculations, after which experiments were performed on an appropriately balanced model, to Brunel's satisfaction.

After Brunel's death in 1859, Froude, enthused by his experimental research into the behaviour of the *Great Eastern*, continued alone to explore the theory of the rolling of ships among waves and, in 1861, presented his conclusions to the Institute of Naval Architects, which had not long been established. For ocean-going steamships, rolling had been a subject of debate for decades and although some naval architects, particularly Russell, openly applauded Froude's work, many eminent men, including academics, had difficulty in accepting Froude's final analysis. William Rankine, the professor of civil engineering and mechanics at the University of Glasgow, was one notable exception. At the next meeting of the Institute, Rankine presented a rigorous mathematical proof of Froude's theory by assuming that ocean waves were of František Gerstner's trochoidal shape, rather than the less complicated sine waves used by Froude to vindicate the results of his experiments. From that moment, Froude, the former railway engineer, country gentleman and mere recreational researcher, emerged as an exceptional and influential figure within the British shipbuilding industry.

Throughout the 1860s Froude became increasingly occupied by the powering of ships and a personal desire to identify the most efficient hull shape, leading to his private research using the ship models, *Raven* and *Swan.* During that exploratory period, adopting the principle that the speed and length of surface waves are related by a square root relationship, Froude devised a formula by which the results of model testing could be used to predict the performance of full-sized ships. Froude's reasoning was that hulls of different size but of exactly the same shape, each travelling at speeds equally proportional to the square root of their respective waterline lengths, would create similar wakes. Using 'LWL' to denote the length of the load waterline, the equation determined by Froude to relate the speeds and waterline lengths of identically shaped hulls can be written as:

$$\frac{v_1}{\sqrt{LWL_1}} = \frac{v_2}{\sqrt{LWL_2}}$$

Applying the formula, a scale model having a waterline length of, say, 4 metres would need to travel at 3 knots to create a wave pattern comparable to that of the full-sized vessel having a waterline length of 100 metres and travelling at, say, 15 knots. Intuitively, the dimensions of the resulting wave patterns would be expected to be proportional to the sizes of the hulls, allowing Froude to expand his theory even further, reasoning that the wave-making resistances of the hulls should therefore be proportional to the hulls' displacements. Friction, the other major component of the total resistance would have to be treated separately.

Realising that recognition of speed-length ratios and his 'law of comparison' could lead to model testing becoming the most effective means of determining the performance and powering of ships, in 1868 Froude began lengthy negotiations with the Admiralty for the establishment of a testing tank facility near Torquay where he had acquired land and already built a mansion and workshops for that purpose. Opposition to Froude's proposal was intense, particularly from Russell who, from his own early experience with models, genuinely doubted the value of model testing and, as a consequence, was obviously riled at having his *Waveline Theory* discredited by Froude's recently completed experiments at Dartmouth. Despite objections, the Admiralty agreed to cover the cost of the tank's construction, its running costs and a salary for Froude's chief assistant, his third son, Robert Edmund Froude. William, forever the gentleman, refused the offer of a salary for himself. Construction of Froude's testing tank facility at Torquay began in 1871 and was operational the following year. Contemporary sources vary in their accounts of the tank's overall dimensions but, as an approximation, it's generally thought to have exceeded 75 metres in length, with a width of about 10 metres and a depth of 3 metres.

Model testing to accurately predict the resistance of new designs had, in the past, always proven to be unreliable. For Froude's testing tank to be successful, it was essential that measurements of the performance of an existing ship be acquired to accurately determine the relationship linking the data abtained from scale models to that of their full-sized counterparts. Two British naval ships were duly placed at Froude's disposal and experiments were organised, primarily to measure the resistance of a typical ship at various displacements over a range of speeds. The ship to be used, HMS *Greyhound*, having a waterline length of about 53 metres and a usual displacement of more than 1000 tonnes, needed to be towed through undisturbed water, clear of the towing vessel's wake, in the same manner that Froude had tested *Raven* and *Swan*. Such an experiment had never previously been undertaken on a large scale, being an expensive and, without question, an exceedingly difficult task that previously, before steamships, had been a physical impossibility. During the experiment *Greyhound* was towed by HMS *Active* which was fitted on one side with a long outrigger, to the end of which the tow line was attached. The horizontal component of the towing force was automatically measured by equipment fitted to *Greyhound's* foredeck and was recorded constantly on a revolving cylinder, as was the speed. For Froude, the results of the experiment were extremely valuable and gave clear indications of the ship's wave-making resistance, frictional resistance and the power lost through inefficiencies in the engine, shafting and propeller. In the tank, using a scale model of the *Greyhound*, Froude was able to gain constructive insights into how the testing of hull shapes should be implemented.

To test models in the tank, Froude had designed a mobile carriage which ran on rails suspended above the water surface. The scale model was located directly beneath the carriage which was fitted with measuring equipment recording both the model's speed and total resistance. To separate the effects of surface friction on a hull's performance from the wave-making resistance, Froude initially used the results of Mark Beaufoy's experiments from the 18th century. Then, by testing planks of varying lengths and roughness in the tank, Froude was able to determine correction factors for frictional resistance that were more accurate, varying according to the length and speed of the model. The models themselves, sometimes up to 6 metres in length, were generally carved from paraffin wax with the aid of a machine invented by Froude to automatically define their shape from the ship's lines plan. Models could be carved in a single working day and used the next, a method so successful that the testing of new designs of British naval ships became almost standard practice. For Froude, the tank offered an opportunity to methodically test typical hull shapes, measuring the effects of altering the waterline length, the beam, the draft, the midship section, the length of middle body, the shape of the forebody, the afterbody or any other feature, all which would be unthinkable at full scale. Froude also continued his investigations into screw propellers and the rolling of ships. Primarily, however, the aim of model testing at Torquay was not necessarily to evolve hull shapes of least resistance but to ascertain the power required to drive already-designed naval ships at desired speeds.

In 1879, just seven years after the testing facility at Torquay became operational, William Froude died suddenly in South Africa, to where he had sailed as an official guest of the Royal Navy. Although a civilian, such was the impact that Froude had on British naval matters in the latter part of his life, that he was buried in South Africa with full naval honours. For his contributions to ship design, Froude had been elected a fellow of the Royal Society of London in 1870, awarded the Royal Medal of the Royal Society in 1876 and, the same year, presented with an honorary degree from the University of Glasgow. Today, Froude, an amateur researcher for much of his adult life, is regarded by many as the most influential individual in the shaping of modern naval architecture.

Froude's period with the testing tank facility at Torquay was all too brief to allow completion of the systematic investigation of the many variations of hull shape that he had initially envisaged. Yet, in those few short years, Froude was able to gain insights into facets of ship design that were otherwise impossible in that era. For instance, while conducting his own experiments in the tank, Froude had run models at scaled speeds well in excess of 100 knots, noting their performance and observing, at close quarters, the anticipated 'dynamic lift' that raised some models above their normal 'at rest' position as speed was increased. But, of all the observations made by Froude, perhaps none was more straightforward, nor more significant from a theoretical viewpoint, than his documentation of the wake made by models of ships at different speeds, a task not easily performed at full scale, especially from the deck of a ship at sea.

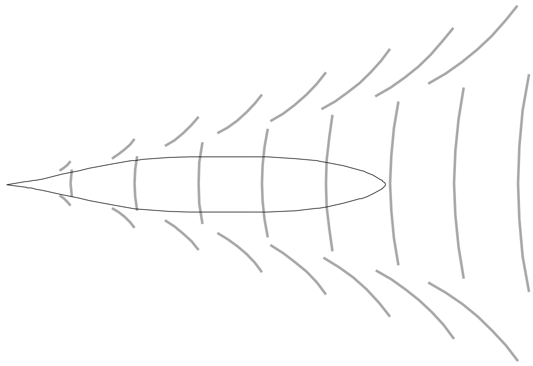


Figure 5-1

Figure 5-1 is a simple sketch of a clean wake, produced by a ship with a waterline length of 100 metres and travelling in deep water at a speed of 10 knots. The familiar bow wave that always forms as a crest at the forward end of the ship's waterline has been omitted for clarity and the lines drawn represent the crests of the other waves within the wake . In reality, additional wake patterns can be formed wherever the vessel's shape deviates from the line of the water's natural flow around the hull, often noticeable at the 'shoulders' where the forebody meets the parallel middle body and at the stern.

As described by Froude:

> *"The inevitably widening form of the ship at her entrance throws off on each side a local, oblique wave of greater or less size according to the speed and obtuseness of the wedge, and these waves form themselves into a series of diverging crests such as we are all familiar with. These waves have peculiar properties. They retain their identical size for a very great distance with but little reduction in magnitude. But the main point is that they become at once disassociated from the vessel, and after becoming fully formed at the bow, they pass clear away into the distant water and produce no further effect on the vessel's resistance.*
>
> *But besides these diverging waves, there is produced by the motion of the vessel another notable series of waves which carry their crests transversely to her line of motion . . . the wave is largest when the crest first appears at the bow, and it reappears again and again as we proceed sternwards . . . but with successively reduced dimensions at each reappearance. That reduction arises thus : In proportion as each individual wave has been longer in existence, its outer end has spread itself further into the undisturbed water on*

either side, and as the total energies of the wave remains the same, the local energy is less and less, and the wave crest as viewed against the side of the ship is constantly diminishing. We see the wave crest is almost at right angles to the ship, but the outer end is slightly deflected sternward from the circumstance that when the wave is entering into undisturbed water its progress is a little retarded, and it has to deflect itself into an oblique position, so that its oblique progress shall enable it exactly to keep pace with the ship . . ."

Froude's description is that of an observer and offers no explanation of how the wake is formed, the reason for its characteristic shape nor why the waves within the wake advance at the same pace as the ship. That there are two distinct wave systems within a ship's wake is familiar to the occupants of any boat that has ever crossed the wake of a larger vessel but the pertinent fact that is often overlooked by casual observers is that the waves within a ship's wake do always keep pace with the ship. Interestingly, to an observer on the shore, the waves within a wake seem highly mobile but, provided the speed and direction of a ship remain steady, to an alert observer onboard or, alternatively, on a carriage towing a scale model in a testing tank, the waves within the wake actually appear static, almost as if frozen into the water surface and being pushed along by the hull. The waves are, of course, moving but are always spaced so that their wavelengths enable them to travel forward at the ship's speed, thereby maintaining their position relative to the ship.

For the ship shown in Figure 5-1, travelling at 10 knots, the transverse waves within the wake require a wavelength of 16.9 metres to keep pace with the ship, calculated from Gerstner's formula for the velocity of a trochoidal wave. If the ship's speed was increased to 15 knots, both wave systems, the divergent bow waves and the transverse waves following the ship, would increase in volume and also spread further apart, thereby increasing their velocities. To keep pace with the ship at 15 knots, the transverse waves would then require a wavelength of 38.1 metres, producing the wake shown in Figure 5-2.

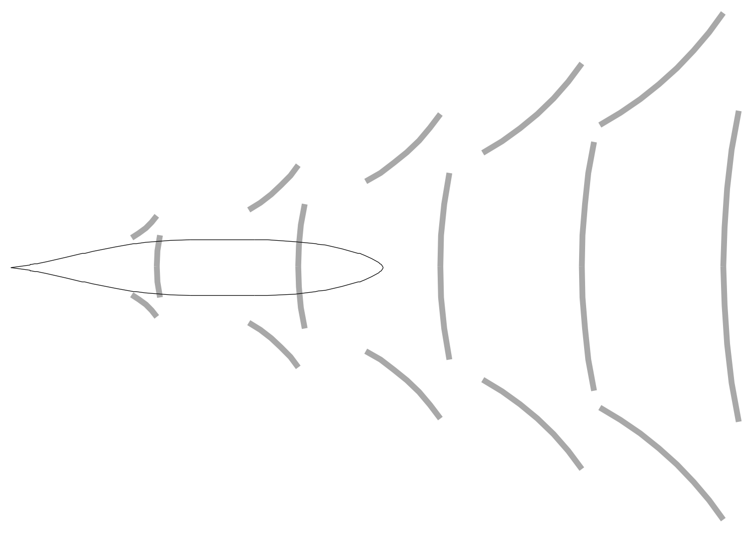


Figure 5-2

At a speed of 20 knots the waves within the wake increase in volume even more and spread even further apart, the transverse waves at that speed requiring a wavelength of 67.7 metres, as shown in Figure 5-3.

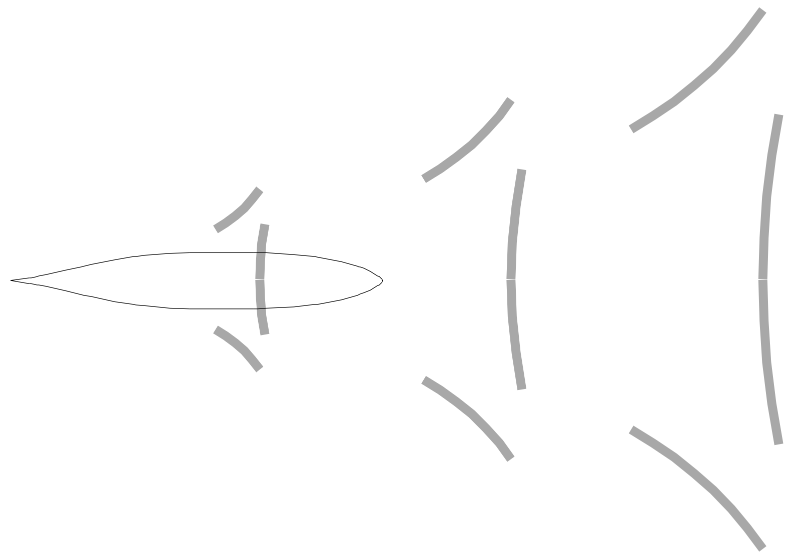


Figure 5-3

If the speed of the ship is increased beyond 20 knots, the transverse waves continue to increase in volume and spread further apart until, at one particular speed, they have a wavelength equal to the length of the ship's waterline. The speed at which that occurs is generally referred to as the ship's 'hull speed', for no particular reason other than that, at that speed, the ship is producing transverse waves with wavelengths equal to the waterline length of the hull. For the ship shown in Figures 5-1 to 5-3, with a waterline length of 100 metres, the velocity at 'hull speed' is 24.3 knots. At speeds higher than 'hull speed' the crest of the leading transverse wave, which then has a wavelength longer than the ship's waterline, would be located well aft of the ship, progressively moving further and further aft as speed is increased.

Besides the actual structure of the wake, Froude's realisation that the waves within the wake of a ship always travel forward at the same speed as the ship, together with the knowledge that the velocity of a wave is proportional to the square root of its wavelength, was the clue that inspired Froude to develop his 'law of comparison' for ships, and models, travelling at the same speeds relative to their waterline lengths or, more precisely, at the same speeds relative to the square root of their waterline lengths. Applying Froude's formula for comparison with identically shaped ships of different sizes, similar wave patterns to those depicted in Figures 5-1 to 5-3, in proportion to the length of the ship, would be produced by a similarly shaped vessel with a waterline length of, say, 25 metres travelling at 5, 7.5, and 10 knots respectively, or by a scale model with a waterline length of, say, 4 metres travelling at 2, 3, and 4 knots.

To graphically demonstrate the relationship between speed and length, Figure 5-4 shows the wake produced by a vessel having a waterline length of 25 metres, travelling at a speed of 10 knots. The wavelength of the transverse waves is 16.9 metres, exactly that of the ship shown in Figure 5-1, which had a waterline length of 100 metres but was also travelling at 10 knots. The wakes appear identical, apart from the volume of water within the waves. In the immediate vicinity of the vessel, however, the wave pattern in Figure 5-4 is completely different to that in Figure 5-1 but, apart from the scale, is exactly that of the ship shown in Figure 5-3, travelling at 20 knots. According to Froude, the vessel in Figure 5-4 can therefore be considered as travelling at the same 'relative speed' as the ship in Figure 5-3.

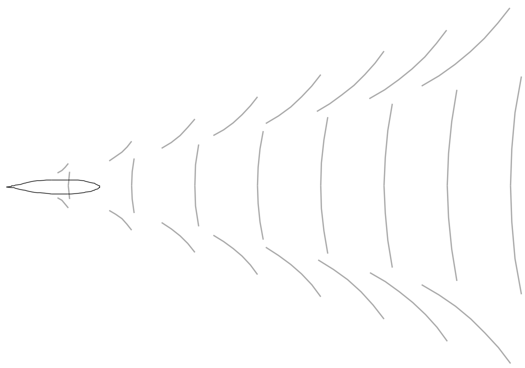


Figure 5-4

By using models, Froude was able to clearly demonstrate that, relative to the ship, the wake is a product of not only the ship's speed but also its waterline length. A 'high' speed for one vessel could be considered a 'low' speed for a ship with a much longer waterline length. To aid comparison between the models that he was testing and the ships they represented, Froude then introduced a new dimension by which to express the measurement of a vessel's speed. Rather than simply referring to the model's and the ship's absolute speeds, Froude instead devised a figure which he termed the 'speed-length ratio', the ratio of a vessel's speed to the square-root of the vessel's waterline length. The speed-length ratio is usually denoted by the letter R and is calculated from the formula:

$$R = \frac{v}{\sqrt{LWL}}$$

According to Froude, if a scaled model and the ship it represents were to travel at different speeds so that they have the same speed-length ratio, the model would be expected to produce a scaled version of the ship's wake, enabling scaled measurements of the ship's resistance.

In the examples given previously, the ship with a waterline length of 100 metres travelling at 20 knots, as shown in Figure 5-3, and the smaller vessel with a waterline length of 25 metres travelling at 10 knots, as shown in Figure 5-4, are, at those speeds, producing similarly proportioned wakes, relative to the size of their hulls. In each case the crest of the first transverse wave is located slightly aft of amidships. If the speeds are measured in knots and the waterline lengths in metres, the speed-length ratios calculate to be 2.0 in each case. Similarly, using the same system of measurement, where the speed is expressed in knots and the waterline length in metres, every vessel travelling at its individual 'hull speed' and producing transverse waves with wavelengths equal to the vessel's waterline length, does so at a speed of $2.43\sqrt{LWL}$, giving a speed-length ratio of 2.43 at that speed. At speeds lower than 'hull speed' R is less than 2.43 and the transverse waves have wavelengths shorter than the vessel's waterline length. Likewise, above 'hull speed' R is greater than 2.43 and the transverse waves have wavelengths longer than the vessel's waterline length. As an aside, in 19th century Britain, speed was measured in knots and waterline lengths in feet, so that Froude determined 'hull speed', for instance, to be equal to $1.34\sqrt{LWL}$. Many texts continue to express speed-length ratios using the same standards of measurement that were used by Froude.

From the very beginning of tank testing at Torquay, Froude's theories proved to be correct. At last, the force required to propel a ship at any given speed could be estimated with reasonable accuracy at the design stage. In principle, the process of deciphering the measured resistance of a scale model so that it could be translated into the resistance of a real ship was reasonably straightforward. For practical purposes, the total resistance of the model, as well as that of the ship, was intentionally assumed to be made up entirely of friction and form resistance which, for simplicity, was assumed to be wholly caused by wave-making. The initial step in the tank testing process was, of course, to construct an accurate scale model of the ship and then tow the model in the tank at a speed relative to the ship's true speed. Measurement of the force required to tow the model revealed the model's total resistance for that speed. Using Froude's experimental data, the frictional component of the model's total resistance for that speed was calculated and deducted from the measured resistance, leaving the remainder as the model's wave-making resistance. Because of the similarity of the wakes of the model and the ship travelling at the same relative speeds, the model's wave-making resistance was reasoned to be scalable, proportional to the respective displacements. Finally, after scaling the model's wave-making resistance upwards to that of the ship, the frictional component of the ship's total resistance was calculated and added, resulting in the estimate of the ship's total resistance for the given speed. In practice, Froude's methods were phenomenally successful.

Using the data accumulated from tank testing, as well as being able to estimate the powering requirements of individual ships, Froude was soon able to demonstrate the general effect that wave-making has on the performance of a ship.

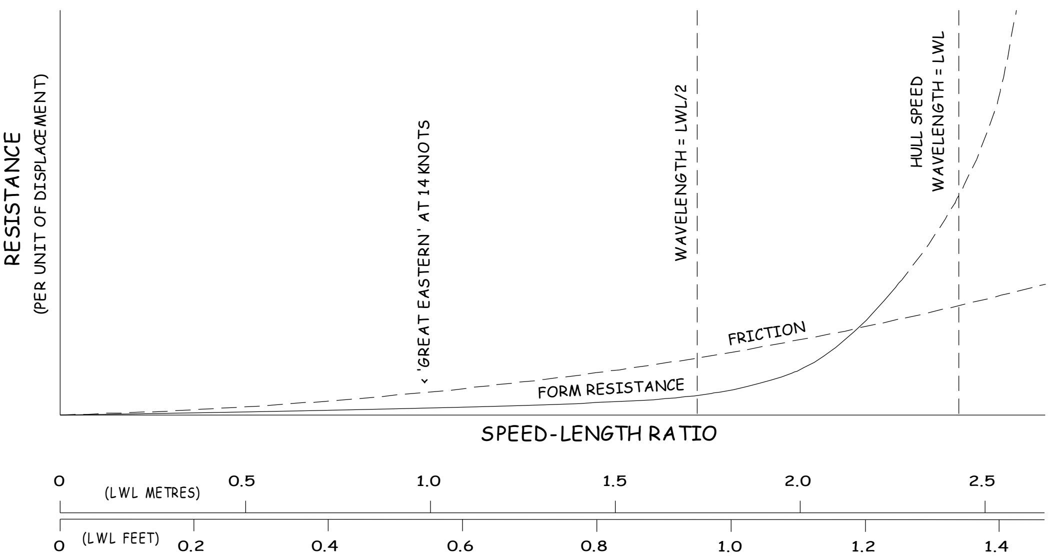


Figure 5-5

As illustrated in Figure 5-5, at very low speed-length ratios form resistance, which is mostly caused by wave-making, is almost negligible compared with the resistance due to friction. As speed is increased the form resistance increases almost uniformly at first, until a speed is reached where the transverse waves have a wavelength about equal to half the waterline length of the ship. From that point, at a speed-length ratio of about R=1.7, the wave-making resistance begins to increase excessively so that, on approaching 'hull speed', wave-making becomes the principal resistance to the ship's forward motion. Beyond 'hull speed' the wave-making resistance becomes so extreme that few ships are capable of being propelled to those 'high' speeds. In a practical world, despite the level of technical capability, the 'need for speed' is usually restrained by economics and, as a consequence, in Froude's era most shipping travelled below the speed where the transverse waves have a wavelength equal to half the waterline length of the ship. Higher speeds, particularly approaching or beyond 'hull speed', were generally the preserve of smaller naval patrol vessels and the like. Putting those speed 'limits' into perspective, at a speed where the transverse waves have a wavelength equal to half the waterline length, a ship about the size of the *Great Eastern,* with a waterline length of approximately 200 metres, would be travelling at about 24 knots. For a ship of that waterline length, 'hull speed' calculates to be about 34 knots. In reality, the *Great Eastern* had a cruising speed of only 14 knots, fast for ships of that era, particularly when compared with sail, but very slow for her waterline length, operating at a speed length ratio of about R=1.0.

For Froude, and the British shipbuilding industry, the success of tank testing meant that the problem of determining the powering of ships had been solved. Despite his outstanding achievement, Froude, again, had been primarily an observer. Although he had ably demonstrated the effects of wave-making and could estimate the resistance that an individual ship would experience due to wave-making at any given speed, Froude did not attempt to explore in more detail the theoretical aspects of resistance. Expected to be extremely complex and, because of Froude's success, possibly redundant, the theory of the resistance caused by wave-making was now incidental and could be deferred, perhaps to be resolved sometime in the future. Froude was not a theorist, his talents were, instead, his remarkable manual skills mingled with his exceptional intuitive approach to solving problems in a practical way, no better exemplified than by his isolation of the components of a ship's resistance so they could be methodically analysed experimentally. Ultimately, Froude was content to leave the purely theoretical aspects of his work to the many eminent mathematical physicists of his day, men such as George Gabriel Stokes, Osborne Reynolds, William Rankine and William Thomson, also known in Britain as Lord Kelvin, all professors at British universities and still familiar names today within the scientific community for their many outstanding achievements. Froude's diligence as a mere observer, however, did inevitably play a significant role in the advancement of general wave theory.

In 1873, after reading a paper by Stokes on the propagation of strong ocean swells generated by distant storms, Froude privately commented to Stokes that the speed of advancement of the swell was not, as thought by Stokes, equal to the speed of the waves themselves. Froude noted that, from his own observations, particularly in the tank but also in the wake of ships, the '*foremost waves are perpetually dying out, as they invade the undisturbed water . . . this means that the train of waves advance faster than the front of the train'.* Russell had reported the identical observation almost three decades earlier but, by and large, had been ignored. Although a common occurrence and plainly visible to any observer studying the approach of the diverging crests of a vessel's wake into calm water, the phenomenon had gone unnoticed by Britain's academia. Stokes investigated Froude's observations and in 1876 wrote to George Biddell Airy to report the result of a theory which he claimed to be new, that the velocity of propagation of a wave group is, in deep water, only half that of the velocity of the individual waves within the group. After publication of the theory, Reynolds took to throwing stones into a pond and the following year provided the explanation of Froude's observation. Immediately, the effects of the 'discovery' of a group velocity, distinct from the phase velocity of the waves within the group, were felt throughout many fields of science and, for ship design theory, none more so than the field of hydrodynamics.

Inspired by the newly gained knowledge of group velocity, Thomson, perhaps Britain's most celebrated scientist of that era and an avid yachtsman who was simply captivated by the beauty and the mystery of the wave pattern created by a ship, embarked on his own personal investigation of the wake. Froude had already described in detail the peculiar characteristics of a ship's wake and in justifying his own interest in the mathematical investigation of such a seemingly trivial matter, Thomson once stated:

> *'The subject of ship waves is certainly one of the most interesting in mathematical science. It possesses a special and intense interest, partly from the difficulty of the problem, and partly from the peculiar complexity of the circumstances concerned in the configuration of the waves.'*

Thomson's notable achievement in this particular study was to come in 1887, a few years after Froude's untimely death, when he managed to derive a mathematical account of the wave pattern within the wake, utilising the theory of group velocity and the relatively new concept of the 'principle of interference'. To eliminate the complexity caused by hulls of different size and shape, Thomson considered, instead, a point source of disturbance which he claimed created a circular pattern of waves at every location along its path, usually shown as in Figure 5-6. By Thomson's reckoning, regardless of the speed of the source, in deep water the pattern of waves within the wake would always be contained within two straight lines, each angled at 19°28' to the direction of travel of the source. Today, the 38°56' V-shaped boundary of a ship's wake is referred to as the 'Kelvin wedge' in his honour. Thomson also calculated that to maintain pace with the ship, the outer ends of the diverging crests formed off the bow would need to travel at a direction of 35°16' to the direction of travel of the source.

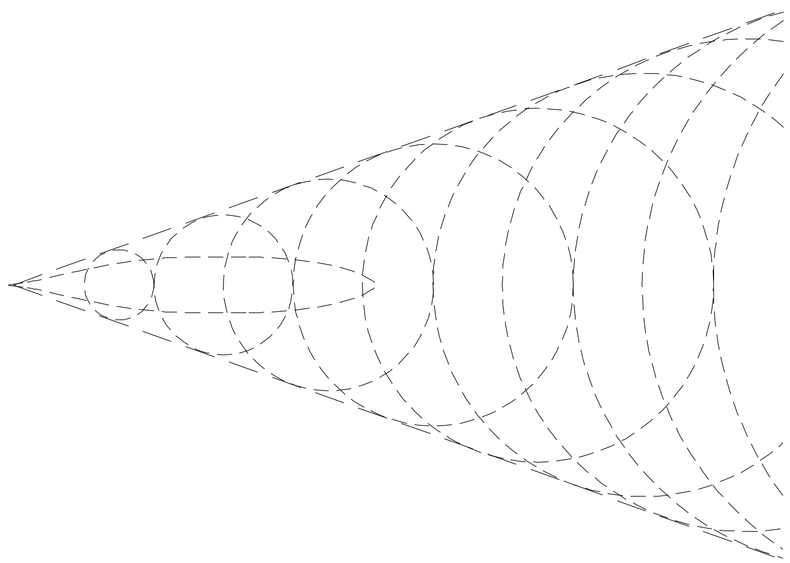


Figure 5-6

In reality, the outer limits of the wake are not easily defined by observation. Interestingly, Froude's sketches of his own observations in the tank seem to indicate that his opinion was that the angle of the wake of a ship is constant except for speeds higher than 'hull speed', above which the wake angle tends to decrease.

Thomson's calculations relied on the observation that the procession of waves behind a ship always appears to be stationary from the perspective of an onboard observer. Although the mathematics used by Thomson to define the wake was, from a layman's perspective, quite complex, his concept of how the wake was formed was extremely simple. At last, after two centuries of scientific investigation, had come the realisation that the wake of a ship could be considered as the combined effect of separate disturbances caused by individual impulses along the ship's path. Each impulse was, in effect, the single splash that British mathematicians had ridiculed and considered undeserving of investigation some three-quarters of a century earlier.

A WAVEFORM THEORY

From the middle period of the 19th century, the development of the steam engine, accompanied by the shift to iron construction and screw propulsion, had begun to split the shipbuilding and boatbuilding industries into two distinct streams. Slowly, but surely, ships for naval and commercial use took on board the new technology and, inevitably, engineering, rather than traditional shipbuilding, was beginning to shape their future. The steamship was on course to becoming a factory-assembled product, of which the hull and engine were but two integral components, resulting in a rising level of specialisation and, as ships grew larger and more expensive, a corresponding rise in the professionalism of all involved. The establishment of the Institute of Naval Architects in Britain during the early 1860s exemplifies the upward change in status of the ship designers of that time. However, from a purely scientific perspective, the flexibility of steamship design and the availability of seemingly ever-increasing power for propulsion had begun to distort the perception of a ship's efficiency. Purpose was becoming paramount, so that rather than encompassing the measure of the ease with which a ship could be driven through the water, the efficiency of a ship was coming to be judged, to a greater extent than ever before, by the capacity to perform specific tasks and, for commercial vessels, the ability to make a profit. Deficiencies in hull shape could, on the whole, be tolerated and overcome, either by installing a more powerful engine and ignoring the consequences or by employing a long, narrow hull to achieve the desired operating speed at a reduced speed-length ratio. The transition to the new engineering approach to ship design was very much accelerated by the success of William Froude's tank testing in the 1870s.

For many smaller vessels, adaptation to steam propulsion and iron construction was, for practical and economic reasons, simply not an option. Whatever their purpose, often commercial fishing, both in protected waters and far offshore, innumerable working vessels continued to be built from timber and driven by sail, unintentionally retaining many of the traditions of earlier times in their design, construction and handling. As might be expected, the working vessels' ongoing links with the past inevitably led to a reputation of irrelevance within the upper echelon of the marine construction industry. However, despite their perceived lack of worth to the future advancement of hull design or construction, it was within the fleets of the smaller working vessels that truly efficient hull shapes continued to evolve. Unlike for steamships, the power available to drive a vessel under sail is not only extremely variable but also limited so that, regardless of a vessel's intended use, ease of propulsion over a range of speeds persisted as a necessary objective of the design process.

For sailing yachts, which had begun to emerge throughout the western world from the working fleets of the 19th century, the ability to slip through the water with a minimum of fuss has, within limits, always been the primary goal. Initially, sailing yachts were indistinguishable from the traditional workboats of their day, modified below decks, perhaps, to provide some of the amenities required when sailing for pleasure. The period of transition during which sailing yacht design and construction completely separated from the workboat industry extended well into the 20th century but, as early as the middle of the 19th century, the development of yachting as a sport had led to the emergence of a new profession, the specialist yacht designer. Gradually, supported by a knowledge of mathematics, physics and, in particular, the ever-developing theories of resistance, creative designers, including talented amateurs, began to transform the traditional workboat hull. Amongst the more noteworthy modifications were the cutback of the traditional deep forefoot to minimise the wetted surface and, in time, the successful use of external ballast, each markedly improving performance, particularly to windward. In a classic work, *Yacht Architecture: a Treatise on the Laws which Govern the Resistance of Bodies Moving* *in Water; Propulsion by Steam and Sail; Yacht Designing; and Yacht Building*, first published in 1885 by Horace Cox, London, the author, Dixon Kemp, a prominent British yacht designer, after quoting Froude's description of the transverse waves within the wake of a ship, highlights the fundamental differences between the steamships and sailing yachts of that era - the shapes of their hulls and, less obviously, the speed-length ratios at which each operated:

> *'. . . It must not be supposed that a number of these transverse waves will be found along the sides of vessels formed like sailing yachts, which have, as compared with steam ships, short and curved middle bodies; but one or more will exist, and be very pronounced at high speeds . . .'*

Most large steamships built for commercial use were almost barge-like in appearance compared with the sailing yachts of the second half of the 19th century. Influenced in design by the concept that the resistance of a hull is comprised of three key elements, the wave-making caused by the forebody, the surface friction of the middle body and the drag of the afterbody, a typical steamship was, in essence, shaped to have a pointed bow, a parallel-sided middle body and a rounded stern. The length of middle body determined both the carrying capacity and, indirectly, the operating speed of the ship. Usually, besides being parallel-sided, the middle body was also straight-keeled and basically rectangular in cross-section, a shape which lent itself to uncomplicated and economical construction in iron. In harbour, a straight middle body with vertical sides was unsurpassed for docking and loading or discharging cargo, and, not surprisingly, that same configuration has largely been retained by commercial shipping to the present day. As noted by Kemp, sailing yachts, in complete contrast, had no parallel middle body whatsoever. Instead, sailing yachts were curved from stem to stern, making use of the natural bend of the timber planking to smooth the flow of water past the hull. From tank testing and his earlier experiments with *Raven* and *Swan*, Froude had ably demonstrated that, overall, rounded, yacht-like shapes actually experienced the least form resistance of all comparable hull types. Although Froude's findings in this regard were not directly applied to commercial shipping, as a result of his experimental work fast naval ships were generally designed with yacht-like curves in mind, minimising the parallel middle body for improved performance, albeit with increased difficulty of construction and added expense.

Compared with vessels under sail, the average speed of a late 19th century steamship was extremely high, even when operating at a low speed-length ratio to reduce wave-making. The *Great Eastern,* as an early example, was capable of cruising at 14 knots, an average speed unattainable by sailing vessels yet well short of her theoretical 'hull speed' of more than 30 knots. As previously illustrated in Figure 5.5, at 14 knots the *Great Eastern's* speed-length ratio was only about R=1.0. By comparison, most sailing yachts of that period were extremely short in length but, although much slower than steamships, were capable of sailing at relatively high speed-length ratios, usually at, or above, R=1.7, a speed at which the transverse waves in the wake have a wavelength about equal to half the waterline length. The technology to harness sufficient wind power to attain such speeds under sail in moderate weather was uncomplicated and the workboat hulls from which sailing yachts evolved had already been honed to near perfection by centuries of trial and error. Despite their efficiency, however, a critical speed for sailing yachts was found to occur near 'hull speed' as the rounded hulls of the sailing yachts produced higher and higher waves within their wakes, the waves described by Kemp as being *'very pronounced'.* No matter how strong the wind nor how able the crews, near 'hull speed' sailing yachts behaved as if trapped within their own wave systems, incapable of going faster.

All surface vessels generate a wake and, as Froude had observed, the transverse waves within the wake of a ship always manage to keep pace with the vessel, adjusting their wavelengths accordingly. Beginning from rest, as a ship gains speed the transverse waves increase their wavelengths until, by definition, at 'hull speed' those wavelengths are equal to the waterline length of the vessel and the crest of the first of the following transverse waves is located near the vessel's stern. Curiously, for the rounded hulls that were typical of the sailing yachts of the late 1800s, at 'hull speed' any further increase in the propulsive force, no matter how great, did not, as might be expected, result in a perceptible increase in boat speed. Unexpectedly, at 'hull speed' the transverse waves within the wakes of even the most streamlined of sailing yachts were observed to suddenly depart from their previous pattern of behaviour. Instead of increasing their wavelengths and travelling faster, as they had done throughout the vessel's lower speed range, on reaching 'hull speed' the transverse waves maintained their 'hull speed' wavelengths and, for reasons not then fully understood, absorbed any additional applied force by increasing their amplitudes, creating higher and higher waves which seemingly trapped the sailing yacht's hulls within their own wave systems, as depicted in Figure 6-1. Not surprisingly, for sailing yachts, unable to develop sufficient power to overcome the energy lost in forming the waves within their wakes, 'hull speed', in the latter part of the 19th century, was more than just a theoretical concept, representing, instead, a very real and seemingly insurmountable barrier to high speed under sail.

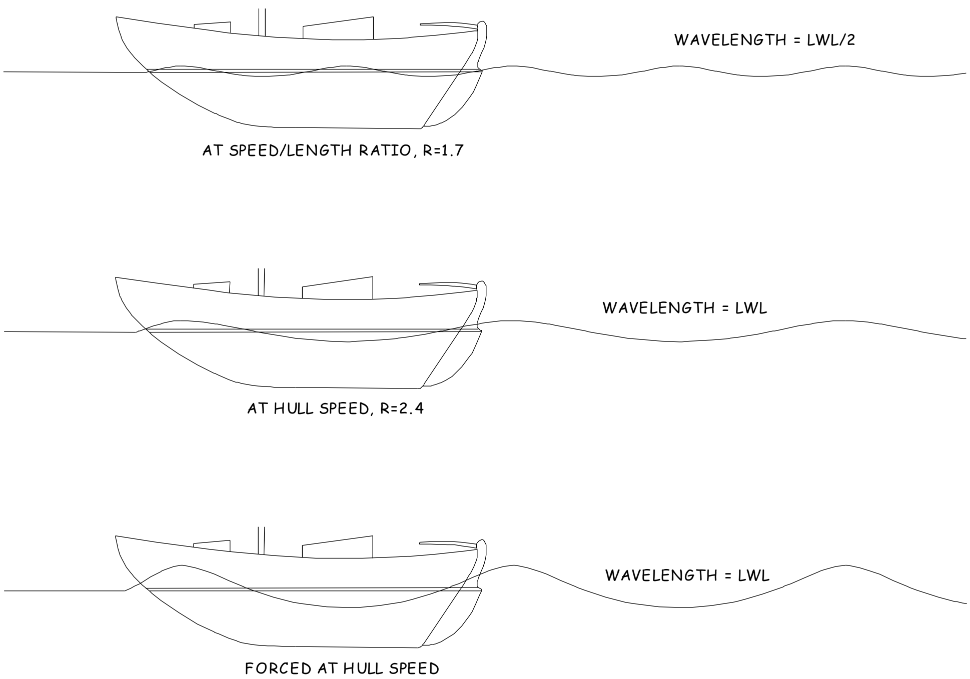


Figure 6-1

For some recreational sailors, the supposed barrier near 'hull speed' was farcical. In North America, for instance, as early as the 1850s, specially developed, lightly built, heavily canvassed, racing boats fitted with centerboards instead of deep keels and crewed by any number of people prepared to act as moveable ballast, were, under favourable conditions, capable of exceeding 'hull speed', but not without the occasional capsize. Such radical boats, being the exception rather than the rule, were, however, generally frowned upon by the yachting establishment and often outlawed from formal competition, stifling their development. Looking beyond their enormous sail areas, other reasons for the extraordinary performances of those types of boat were, for the most part, disregarded, strengthening the ongoing belief that the only way to achieve higher speeds in 'proper' racing yachts was to increase the sail area to displacement ratio, in other words, add more sail and reduce the overall weight. The dilemma for sailing yacht designers was that the capacity to carry sail is dependent on the beam and the stability of the hull, but, on the other hand, a wider and heavier hull needs increased sail area to push the hull through the water. Unlike for steamships, the driving power and the form of the hull of any sailing vessel could not be treated independently, leaving only one alternative for improvement apart from the rig, to design the hull to move through the water with an absolute minimum of resistance so that speeds approaching 'hull speed' could at least be achieved more easily. For that purpose, in 1877, a decade before William Thomson's portrayal of how a vessel's wake is formed, a Norwegian designer, Colin Archer, had put forward a fresh interpretation of old ideas, a variation of John Scott Russell's *Waveline Theory* which, he believed, could minimise the resistance caused by the formation of waves.

Archer, the second youngest of thirteen children, nine boys and four girls, was born in the town of Larvik, Norway in 1832, his family having emigrated from Scotland just seven years earlier. William Archer, Colin's father, had been the junior partner of *Charles Archer & Son,* trading as ship owners and timber merchants out of the small town of Newburgh on the Firth of Tay, on Scotland's east coast, until their small vessels were sunk by privateers. The family firm was almost ruined and, with the country in depression, William chose to focus on Norway and the opportunities there to export timber. But the economic recovery was short-lived, putting pressure on the family's finances, resulting, instead, in William setting up a small lobster trade with England. Times were bad and, in the words of a descendant, *'here they brought up their large family in plain living and straight thinking; and from there they sent out many tall sons to seek their fortunes in the world, chiefly in Australia, where several members of the mother's clan were already, in the eighteen thirties, prosperously established.'* Soon after Colin's birth, the exodus began, one son arriving in Sydney, Australia, in 1834, joined by two of his brothers four years later. Having little capital, at first they worked for others and took their wages in sheep, but the continent was still largely unexplored by Europeans and, in 1838, the Archer brothers headed north, establishing their own sheep station of about 50,000 hectares west of Brisbane, which, at the time, was little more than a penal settlement. Over the next decade or so, other brothers came and went, all the while continuing their exploration further and further north for country more suitable for grazing sheep and cattle. Colin, meanwhile, attended school in Larvik, excelling in mathematics, and enjoyed his boyhood on the family's idyllic property, a former home to the customs officers of the town, surrounded by water on three sides and destined to become the site of Archer's boatbuilding venture in the years to come. At the completion of his schooling Colin began an apprenticeship as a carpenter in a local shipyard and attended evening classes, but Australia beckoned and, after only eighteen months, he boarded a ship bound for Panama to join his brothers.

In 1852, after detouring to California to meet up with a brother lured by the gold rush and then to the Hawaiian Islands to meet another working a plantation, Colin Archer finally arrived in Australia as a young 20 year old. The following year, with Colin's help, the family partnership in Australia marked out more than 1800 square kilometres of good land in the valley of the Fitzroy River which they had recently discovered and named, becoming the first Europeans to settle where the town of Rockhampton, Queensland, now stands. Not only was the land ideal for grazing, having more than 100 kilometres of river frontage would allow the use of sea transport for taking out produce and bringing in stores. Throughout 1854 Colin helped to clear and prepare the land and during the next year the Archers set out from the south with several thousand sheep, bullock teams, horses and all the men and equipment needed to establish a home in new country. Progress was slow and five or six weeks were spent in reaching the station. Colin, instead, travelled to the town of Maryborough to acquire a small vessel to sail up the coast with provisions for the overland party, becoming the first person to sail up the Fitzroy River to the present site of Rockhampton, named after the rocks at the head of navigation. A bronze statue of a horse and rider now stands on the riverbank in the town, commemorating the meeting of Charles Archer, Colin's eldest brother, with the supply vessel. In less than six months the first wool clip was dispatched by boat, taken south along the coast to the town of Gladstone for shipment to Sydney. The sheep station proved to be a success, land values soared and six years later, in 1861, Colin Archer, then almost 30 years of age, financially independent and a skilled businessman, returned to his birthplace in Norway with the intention of starting his own enterprise, as a boat designer and builder.

Archer entered the boatbuilding industry at a time when boat design and construction were still very much influenced by traditional methods. Despite a high level of craftsmanship, visible in the construction of any planked timber vessel, and an extensive knowledge of matters associated with their trade, many boatbuilders were, in fact, barely literate and generally distrusted theory, for good reason. Existing designs had evolved over decades, if not centuries, and for boatbuilders, the shape of a hull or the configuration of a rig was not a matter for some new-fangled mathematical formula but was, instead, decided by means of conservative principles that had been derived by trial and error and passed from master to pupil over a lifetime of practical experience. Archer's entry into the industry was, therefore, somewhat unusual, having served only a fraction of the normal seven years or so of a shipwright's apprenticeship. However, in his favour, his family did have a background in the timber industry and commercial shipping. As well, Archer, himself, had grown up amongst the fishing boats and pilot boats of the Larvik Fiord, absorbing knowledge of their characteristics, and, as a young man, had the opportunity to broaden that knowledge, sailing the Queensland coast. As somewhat of an outsider to the boatbuilding industry, Archer also realised that existing traditional designs had deficiencies which resulted in an unacceptable loss of life of skilled fishermen and pilots, year after year. From Archer's perspective, any number of skilled shipwrights could be employed in his boatyard but it was the design process that had captured his imagination and it was as a designer that he wished to employ his own talents.

Not having any formal training, Archer first needed to educate himself on the theory of design and began studying all the available literature on practical and theoretical shipbuilding including, amongst other works, the writings of Russell, whose controversial *Waveline Theory* was on the verge of being discredited by Froude. After having gained sufficient confidence in his own capabilities, in 1867 Archer designed and built a sailing yacht for his own use and, with subsequent orders, the Archer name soon became synonymous with extremely seaworthy double-ended sailing yachts, simply and robustly built in the Scandanavian tradition. A few years later, in the early 1870s, following the loss at sea of a number of pilot boats, Archer moved into pilot boat construction also, accepting a challenge to improve their design. He increased the displacement to provide headroom below, moved the ballast to the keel and refined the bows to improve the vessels' windward ability in heavy weather and to reduce pounding when hove-to. Before long, Archer's version of the offshore pilot boat, typically about 15 metres in length, became the standard type for pilot boats serving along the entire Norwegian coastline. Being an 'outsider', not unduly prejudiced by tradition, had proven to be an adavantage and, by the mid 1870s, Archer's design successes were beginning to earn him a reputation abroad. In 1877, after personally experiencing the void left by the inadequacy of other design theories, Archer projected himself onto the international stage, submitting his own innovative idea to reduce wave-making resistance. Archer's *Waveform Theory*, distinct from any theories involving the 'lines' of a hull but not necessarily in conflict with them, was published in London in 1878, the year before Froude's death, and was so simple in concept that, on reflection, it seems incredible the idea hadn't been considered decades earlier.

Arguments against Russell's *Waveline Theory* had, over the years, sometimes included the notion that, within reason, the longitudinal distribution of a hull's displacement is far more influential than the actual lines of a hull in determining the resistance of a vessel as it moves through the water. Froude, himself, agreed that the ratio of the growth of the displacement in the forebody and the decrease in the afterbody definitely influence resistance and, throughout his tank testing, always carefully recorded a model's 'curve of displacement', the curve of the cross-sectional areas of the underwater portion of the hull which displays how the underwater volume is distributed along the hull's length. A curve of displacement rising sharply forward, for example, is an obvious indication of a bluff bow that will push water before it, undoubtedly increasing the wave-making resistance, but the question as to what might be the optimum shape for the curve of displacement, if any, remained unanswered. For his solution, Archer began with the assumption that Russell's early experimental work on waves was correct, that is, the disturbance caused on the surface of the water by the forebody of a moving vessel takes the form of a solitary wave and that the wave following the vessel is an ordinary oscillating wave of the sea, trochoidal in shape. In describing his theory, Archer first gave an account of Russell's conclusions:

> *'It is stated that the water which is excavated by a ship when she is moved through the fluid is carried away to a distance (not the identical particles of water in her track, but a corresponding mass), by a solitary carrier-wave, or wave of translation, and that the cavity formed by her largest section is filled up by a wave of the second order, or a common oscillating wave of the sea; therefore the entrance or fore body should correspond in length and form to the length and form of the carrier wave, travelling at the speed of the ship, and the run, or after body to the length and shape of the front of an oscillating wave. If so formed the ship will meet with a minimum of resistance in her progress.'*

Then, taking a broader view than Russell of how the waves caused by a moving vessel are formed, Archer simply contended that to 'fit' a hull to the those waves it was not wave 'lines' that were required, as determined by Russell, but, instead, a wave 'form' in the curve of displacement, a curve of versed sines for the forebody and a trochoidal curve for the afterbody, as shown in Figure 6-2.

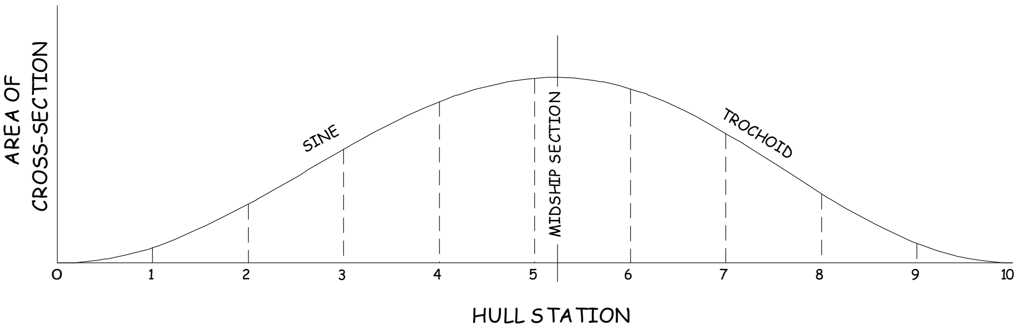


Figure 6-2

Anticipating opposition to his theory, Archer continued, dealing with the shape of the forebody first:

> *'An advocate of wave water lines might object that, since the area of the vertical sections are determined by the form of the water lines — these lines determining the lengths of the ordinates by which the vertical areas are computed — and since a certain progression of these areas is essential, therefore a certain curve for the water lines is equally essential . . . but the same result may be brought about in a great variety of ways. It is important to remember that the shape of the vertical sections is, within reasonable bounds, immaterial as far as the wave theory is concerned. The same liberty claimed in this respect for the midship section, applies with equal justice to any other section. That which is the material point is, that each cross section shall have the exact area which its station in the ship assigns to it, since, if this is the case, the water will be displaced precisely in the required progression. We are, therefore, at liberty to alter the shape of the sections of our fore-body at pleasure.'*

Again, no doubt anticipating criticism, particularly from supporters of the streamline theory, Archer was careful to add:

> *'It is always understood that due care is taken to ensure a 'fair' body, and to avoid abrupt curves.'*

Archer then resumed his explanation of the *Waveform Theory* with a more detailed description of its application to the afterbody:

> *'The form of the after body is required by the wave theory to be such that an oscillating wave following a vessel with her own velocity shall at every point just fill the vacancy left by each succeeding section, and no more. This is an essential condition. Suppose, for instance, that at a given distance from the initial point of the wave — which will be at the largest section of the ship — a section of the said wave has attained to one-fourth the area of its greatest section. It is then required that room shall have been provided for sufficient water to form this area of cross section. A little farther aft the sectional area of the wave will have reached one-third of its greatest area and room must by the time this is reached have been provided for this addition to the area of the wave section. The area of any vertical section in the afterbody must therefore be the area of the midship section less the area which the swell of the wave supplies at that stage of its formation. The wave is not perfectly formed until the stern end of water line is reached. Therefore, what is wanting in volume at any point must be made up by the body of the vessel at that point. As the water flows to form the following wave from every point from which it has free access, we need not have the curve all one way.'*

Finally, Archer offered a formula by which the radius of the trochoidal curve representing the areas of the cross-sections of the afterbody could be calculated:

$$r = \frac{4D}{\pi M} - \frac{2L}{\pi}$$

The derivation of the formula is straightforward. In Archer's equation, D represents the displacement, which is the total area under the curve, L the load waterline length, and M the area of the largest cross-section, generally referred to as the midships area.

If the length of the forebody is, say, l_1 and the curve of areas of the forebody is represented by a sine curve, then the area under the forebody curve becomes:

$$A_{forebody} = \frac{1}{2} l_1 \, 2r$$

Similarly, if the length of the afterbody is l_2 and the curve of areas of the afterbody is represented by a trochoidal curve, then the area under the afterbody curve becomes:

$$A_{afterbody} = \pi R r + \frac{\pi r^2}{2}$$

The volume of the forebody is assumed to be equal to that of the afterbody, therefore:

$$\frac{1}{2} l_1 \, 2r = \pi R r + \frac{\pi r^2}{2}$$

$$l_1 = \pi R + \frac{\pi r}{2}$$

$$l_1 = l_2 + \frac{\pi r}{2}$$

This result means that the location of the midship section can be determined and is dependent on the value of r.

Using the known values D, representing the displacement of the vessel, and L, the load waterline length,

$$L = l_1 + l_2 = 2\pi R + \frac{\pi r}{2}$$

$$M = \frac{D}{l_1} = \frac{D}{\pi R + \frac{\pi r}{2}} = \frac{2D}{L + \frac{\pi r}{2}}$$

from which Archer’s value of r is derived:

$$r = \frac{4D}{\pi M} - \frac{2L}{\pi}$$

For a given waterline length and displacement, the selection of M, the area of the largest cross-section, determines both the location of the midship area and the shape of the curve of the cross-sectional areas of the underwater portion of the hull. At its maximum practical value of 2D/L, the location of M calculates to be at the mid-point of the waterline and the area curve of the afterbody then takes on the form of a curve of versed sines, matching that of the forebody. For smaller values of M, the location of the midship section moves aft and the area curve of the afterbody fills out to a fatter trochoidal shape. In practice, for the sake of simplicity, use of the trochoidal curve was often avoided and, after manually selecting a location for the midship section, usually just aft of the mid-point of the waterline, a curve of versed sines compressed into a shorter length of the afterbody was substituted for the trochoid with acceptable results.

In experienced hands, Archer's *Waveform Theory* was used to produce efficient sailing yachts and was particularly suitable to the wholesome designs on which Archer had already built his reputation. Unfortunately, in practice, the *Waveform Theory*, as presented by Archer, had only limited application, producing hulls which were particularly efficient at the usual sailing speeds of about speed-length ratio R=1.7. Although sailing yachts built to Archer's *Waveform Theory* were capable of higher speeds, they still remained trapped by the 'hull speed' barrier and, as a consequence, Archer's theory had little relevance to high-speed steam-powered vessels. For sailing yacht designs, Archer's *Waveform Theory* proved useful until the early years of the 20th century when practical experience and results of tank testing led to a gradual decline in its popularity. The significance of Archer's *Waveform Theory* is that it managed to divert attention from the natural tendency to regard 'lines' as the key to determining the most efficient shape for a hull to, instead, the importance of the distribution of a hull's underwater volume. Today, in various forms, the 'curve of areas' remains a most important tool for designers of all types of vessels.

Colin Archer died at Larvik in 1921, aged 89. Over almost a fifty year working life as a boat designer and builder, until his retirement in 1909, Archer was responsible for the design and construction of more than 200 vessels, including sailing yachts, pilot boats, rescue vessels and sailing ships, all the while intent on improving their design, particularly their seaworthiness. Perhaps the most famous of all his creations was that of the unusual three-masted topsail schooner, the 39 metre *Fram*, designed and built by Archer at Larvik for Norwegian polar exploration in the Arctic and Antarctic. Archer's sailing yacht designs are still highly regarded and, over the years, several variations have been drawn by other eminent designers, resulting in 'Colin Archer' type sailing yachts becoming legendary amongst ocean-cruising yachtsmen around the globe.

A RETURN TO EVOLUTION

By the close of the 19th century, the wholly scientific approach to ship design had made few inroads into determining, mathematically, the most efficient shape for a hull. In stark contrast, the relevance of model testing, using the methods devised by William Froude in the early 1870s, was, by then, well and truly established. Interestingly, as a show of support for a scientific approach, in 1876, the year before Colin Archer put forward his *Waveform Theory*, Froude had presented to the Institute of Naval Architects in Britain a paper titled *'Fundamental Principles of the Resistance of Ships'*, a commentary on the application of scientific knowledge to the question of a ship's resistance to forward motion. In his preamble Froude drew attention to the many formulae that had been *'constructed by mathematicians'* in the past to account for the resistance experienced by a body moving through a fluid but which, in practice, had proven to be unsound:

> *'These formulae were not all alike, but they were mostly based on the supposition that the entire forward part of the body had to exert pressure to give the* (fluid) *particles motion outwards, and that the entire after-part had to exert suction to give them motion inwards, and that there was, in fact, what is termed plus pressure throughout the head end of the body, and minus pressure or partial vacuum throughout the tail end. And as it seemed that the number of particles which would have to be thus dealt with would depend on the area of maximum cross section of the body, or area of ship's way, as it was sometimes termed, the resistance was supposed to bear an essential proportion to the midship section of the ship. This idea has sometimes been emphatically embodied in the proposition that the work a ship has to do in performing a given voyage is to excavate in the surface of the sea, from port to port, a canal the cross section of which is the same as the midship section of the ship.*
>
> *This theory of resistance was at first sight natural and reasonable; it was generally admitted for many years to be the only practicable theory, and was embodied in all the most approved text-books on hydraulics and naval architecture. But when the theory of streamlines was brought to bear upon the question, then it was discovered that the reactions, which the inertia of the fluid would cause against the surface of the body moving through it, and which were supposed to constitute the resistance, arranged themselves in a totally different manner from what had previously been supposed, and that, therefore, the old way of estimating their total effect upon the ship was fundamentally wrong.'*

Guided by the opinions of eminent British physicists, including William Rankine, George Stokes, Osborne Reynolds and William Thomson, Froude then devoted '*considerable space*' to denouncing the old approach. Newton's original concept, that form resistance is the result of the direct impact of a moving body on the fluid particles located along its path, was dutifully reasoned to be obsolete, so too the proportionality of that resistance to the body's maximum cross-sectional area. As a substitute for Newton's ideas, Froude offered a brief, elementary outline of the streamline theory and its relevance to ship design. Curiously, though, in his closing remarks, after emphasising the foolishness of ignoring the contribution of modern science to ship design, Froude, at the very last, disclosed an underlying uneasiness in his own mind of the rejection of the old approach:

> *'In conclusion, let me again insist, and with the greatest urgency, on the hopeless futility of any attempt to theorise on goodness of form in ships, except under the strong and*

entirely new light which the doctrine of streamlines throws on it.

It is, I repeat, a simple fact that the whole framework of thought by which the search for improved forms is commonly directed consists of ideas which, if the doctrine of streamlines is true, are absolutely delusive and misleading. And real improvements are not seldom attributed to the guidance of those very ideas which I am characterising as delusive, while in reality those improvements are the fruit of painstaking but incorrectly rationalised experience.

I am but insisting on views which the highest mathematicians of the day have established irrefutably; and my work has been to appreciate and adapt these views when presented to me.

No one is more alive than myself to the plausibility of the unsound views against which I am contending; but it is for the very reason that they are so plausible that it is necessary to protest against them so earnestly; and I hope that in protesting thus I shall not be regarded as assuming too dogmatic a tone.

In truth, it is a protest of scepticism, not of dogmatism; for I do not profess to direct any one how to find his way straight to the form of least resistance. For the present we can but feel our way cautiously towards it by careful trials, using only the improved ideas which the stream-line theory supplies, as safeguards against attributing this or that result to irrelevant or rather non-existing causes.'

Evidently, for Froude, the conflict between science and intuition had not been adequately resolved.

Tank testing, Froude's compromise solution to a predicament which science had shown it was not sufficiently expert to unravel, had already proven to be invaluable. The significance of Froude's achievement was that, after testing a model of an already-designed ship, the engine power required to overcome the resistance to drive the ship at a given speed could be calculated prior to construction. No accurate alternative method to determine the powering requirements of a ship existed and Froude's results had been so impressive that as early as 1874 the Dutch had constructed, at Amsterdam, a testing tank similar to Froude's, followed in 1877 by the French, at Brest. In 1883, a Scottish shipbuilder, William Denny, completed the construction of the world's first civilian testing tank at his Dumbarton shipyard near Glasgow and, as a sign of things to come, four years later installed a wave-making machine to simulate ocean waves. Meanwhile, Robert Froude, after his father's death in 1879, continued investigations at Torquay until 1886, at which time he headed a shift to Haslar, near Portsmouth, where a more extensive and better equipped facility had been established by the Admiralty, setting a pattern of improvement in testing tank facilities that would continue beyond the next century. The Haslar tank, at about 120 metres in length, was more than fifty per cent longer than William Froude's original. Similar facilities were soon established with Robert Froude's assistance at La Spezia, Italy, in 1889 and at St Petersburg, Russia, in 1892. In the United States of America a towing tank of some 140 metres in length, designed by David Watson Taylor, became operational in 1898 at a naval establishment in Washington. Obviously, from the viewpoint of the shipbuilding industry, the wholly scientific approach to ship design had foundered.

The quest by scholars to derive a universal mathematical formula for Isaac Newton's imagined 'solid of least resistance', particularly with regard to the application of the concept to hull design, had, for the most part, already been put to rest. Two centuries of investigation, involving, at times, some of the world's most talented mathematicians and physicists, had failed to deliver an adequate solution to what was initially anticipated to be a straightforward problem. Hydrodynamics, the study of fluid mechanics which had formally introduced the concept of streamlines and was expected to resolve the issue of the resistance of ships, had become, instead, an extremely complex topic in its own right, seemingly producing more questions than answers about fluid behaviour. Moreover, similar trends had occurred in all branches of science so that towards the end of the 19th century even the most gifted intellectuals were being overwhelmed by the escalation of knowledge, or lack of it. Inevitably, within academia, individual scholars were being confronted with little alternative but to focus their attention on increasingly limited subject matters and, as a consequence, scientific research in all fields was, by necessity, becoming more institutionalised than in the past and, unavoidably, more and more expensive. Under those circumstances, within the discipline of naval architecture the most realistic opportunity for the advancement of scientific knowledge was deemed by academics to be not in the futile pursuit of theory but in the refinement of Froude's tank testing process. Remarkably, the wheel had turned a full circle and, as the world approached the 20th century, ship design was experiencing a wholesale return to the evolutionary methods of the past, albeit in a much more sophisticated style.

Steamships, during the last decades of the 19th century, had improved markedly in both design and performance, due in no small part to the introduction of tank testing and the development of more efficient steam engines. By the beginning of the 20th century, ocean liners, for example, were not too dissimilar in appearance and performance from the cruise ships of today. Gone were the paddle wheels and the auxiliary sails of the *Great Eastern*, replaced by powerful steam engines driving screw propellers, often installed in multiple configurations. Average cruising speeds of the fastest liners had exceeded 20 knots by 1890 and continued to improve, culminating in a record crossing of the Atlantic by the British-built *Mauretania* in 1909 at an average speed of about 26 knots, a record that would stand for two decades. In practice, though, steamships for commercial use were generally designed to operate at speeds less than the maximum that engines could provide, not only to improve passenger comfort but, primarily, to reduce the enormous fuel consumption. The ill-fated *Titanic*, for example, launched in 1911 to commercially rival the *Mauretania*, recorded a top speed of about 23 knots during trials but was intentionally designed to cruise at only 21 knots, a pace at which she suffered less vibration from her engines and consumed only 600 tons of coal per day, compared with the *Mauretania's* 1000 tons at high speed. Cargo ships were generally slower but, in contrast, the fastest warship of that era, the German navy's *Vaterland*, freed from the economic constraints of commercially operated vessels, was capable of 30 knots at full speed which, inevitably, because of fuel consumption, was unsustainable over long distances. Nevertheless, the average speeds of ocean-going steamships in the early decades of the 20th century were high, even by modern standards, and far in excess of speeds achievable under sail. However, despite those unprecedented speeds, the speed-length ratios at which ocean-going ships operated remained relatively low. The warship *Vaterland*, for example, operated at a speed-length ratio of about R=1.8 at top speed, the liner *Mauretania* at about R=1.7 while cruising and the *Titanic* at only R=1.3, each well below 'hull speed', R=2.4. Tank testing had, no doubt, improved efficiency but the higher speeds of these ships were, for the most part, due to increased waterline lengths and the availability of more powerful engines. By boating standards the ships were incredibly long, the *Vaterland* having an overall length of about 276 metres, the *Mauretania,* 241 metres, and the *Titanic*, 269 metres.

Throughout the 20th century, tank testing proved to be an indispensable aid to the design of ships, a role which seems likely to continue for the foreseeable future. Towing tanks of more than half a kilometre in length are now not uncommon and specialist testing tank facilities have been developed to investigate particular aspects of ship design such as sea-keeping, manoeuvrability in confined spaces and, in ice tanks, the efficiency of ice breaking. In addition to a ship's resistance, other design factors such as the streamline flow around a ship's hull, the efficiency of a ship's propellers and rudders, the profiles of the waves within a ship's wake and the ship's angle of trim can each be observed and measured in a testing tank at comparatively small cost, allowing the performance of a design to be predicted within a few percent. Today, few, if any, naval or commercial ships are built without a prior comprehensive study of their performance using models. However, despite the steady improvement in the efficiency of large ocean-going ships, the sustainable speeds at which modern ships operate tend to mask the fact that the speed-length ratios of conventional ships have barely increased, if at all, since Froude and, notwithstanding their high speeds, conventional ships continue to operate at speeds much lower than 'hull speed'.

The fastest ocean liner of all time is on record as being the steamship *United States*, launched at Newport News, Virginia, USA, in 1951. Capable of exceeding 38 knots at full speed, the *United States* crossed the Atlantic at an average speed of about 36 knots on her maiden voyage and in later service consistently cruised at 'only' 31 knots. Slightly longer than 300 metres overall, the *United States* achieved maximum speed at a speed-length ratio of about R=2.2, a little below 'hull speed', dropping to about R=1.8 while cruising. In comparison, a more recent design, the British cruise ship, *Queen Mary II*, launched in 2003 and considered to be a modern version of an ocean liner, has an official top speed of 28 knots and a cruising speed of 26 knots, which, because of an overall length of 345 metres, are achieved at speed-length ratios of about R=1.5 and R=1.4 respectively. The majority of modern cruise ships operate at speed-length ratios that are even lower. Naval ships are, of course, much faster but, although their performance statistics are not often available for analysis, the trends are similar. To this day, some of the fastest conventional warships ever built are considered to have been those of the French navy's *Malin* class destroyers, launched in the 1930s. With an overall length of about 132 metres and a top speed in excess of 40 knots, these ships could operate at a speed-length ratio approaching R=4.0, well above 'hull speed' but, again, such high speeds were not sustainable. Even at a lower speed of 34 knots, the destroyers' normal cruising range of about 6,500 kilometres at 17 knots, was reduced to little more than 1000 kilometres due to fuel consumption. Clearly, given sufficient power, conventional ocean-going ships can be pushed to high speeds but, like the sailing yachts of Froude's era, for reasons of practicality ships, too, are effectively trapped by the 'hull speed' barrier.

As a 'trial and error' technique to improve the performance of ships, tank testing, if assessed solely on the stagnant speed-length ratios at which conventional ships have operated for more than a century, could be wrongly judged to have had limited success since its inception, even though the benefits of tank testing have probably far exceeded the wildest expectations of those who encouraged its initial development. Essentially, the repetitive tank testing process is one of cautious observation. Inventiveness is far less predictable and often materialises from an unexpected source, there being no more apt example in maritime history, surely, than a retired railway engineer's use of models to successfully determine the resistance of ships in the 1870s. Less surprisingly, after tank testing became more closely allied to academia in the late 19th century, the advances in hull design that have subsequently emerged have not always evolved from the orderly, methodical approach envisaged by the establishment. Often it has been in the hands-on 'hit or miss' world of recreational boating that hull design has taken a new direction.

The 20th century, from the very first decade, was one of unprecedented change in the way that most people lived. Over the span of an average lifetime, advances in science and technology catapulted the world from a horse and buggy era to one of international air travel and the beginning of space exploration. Setting the scene for the change of pace that was to come, two bicycle manufacturers from Dayton, Ohio, USA, brothers Wilbur and Orville Wright, after a lengthy period of experimentation, and ridicule, established, beyond all doubt, the practicality of powered flight. Giving the first public demonstrations of their invention almost simultaneously on both sides of the Atlantic, in France and the USA, Wilbur flew first, undertaking a few short flights in 1908 near Le Mans, France. Within a month, exhibiting a similar aircraft at Fort Myer in Virginia, USA, Orville made the world's first hour-long flight, lasting 62 minutes and 15 seconds. In the heady days between those inaugural flights, the first of Henry Ford's famous Model T automobiles rolled off the assembly line at Detroit, Michigan, USA. While steamships were striving to break the record for the fastest trans-Atlantic voyage, a revolution in transport was in full swing, enabled by a lighter, more compact and much more practical substitute for the steam engine, the internal combustion engine.

The first use of an internal combustion engine in a boat is considered to have taken place in 1886, when a German mechanical engineer, Gottlieb Daimler, and his business partner, industrial designer Wilhelm Maybech, installed a single cylinder, low powered engine of their own design in a 4.5 metre launch. Subsequent improvements to the internal combustion engine were rapid and by the turn of the century a number of engine manufacturers had begun to turn their attention to the marine industry, where there was obvious potential. At that time, the vast number of small commercial vessels around the world were still operating under sail, unsuitable for conversion to steam. Nevertheless, despite the obvious advantages of the new type of engine, the inevitable switch from sail to power was not immediate. Engines were bulky and expensive, with many teething problems to be resolved, including the basic prerequisites of safety, reliability and ongoing maintenance. For some wealthy enthusiasts, enticed by the novel prospect of racing boats at high speed, overcoming those shortcomings was viewed, instead, as an exciting challenge in which they were prepared to invest, not always for monetary gain. So it was that within the first few years of the 20th century, before the internal combustion engine was accepted as a substitute for sail in small commercial vessels, the sport of powerboat racing was established on both sides of the Atlantic.

Ideas on how to achieve high speed on the water by overcoming the 'hull speed' barrier were far from new amongst powerboat devotees. The predictable line of thought was to simply increase the power to weight ratio of conventionally shaped hulls to smash through the 'hull speed' barrier with brute strength. Charles Parsons, the British inventor of the steam turbine engine, had already shown how to do just that, better than most. In 1894 Parsons had commissioned the design and construction of a vessel into which one of his steam turbine engines was fitted, primarily for demonstration purposes. Appropriately named *Turbinia*, the vessel was some 30 metres in length, with a beam of only 2.7 metres and a displacement of about 44 tons. After cavitation problems with the high-speed propellers were resolved, *Turbinia* firmly established the superiority of the powerful and efficient steam turbine over the reciprocating steam engine by achieving a top speed of 34 knots, a speed-length ratio of about R=6.0. At the other end of the design spectrum, existing mainly in the imagination, was the 'hydroplane', a flat-bottomed craft intended to skim across the top of the water at high speed, similar in principle to a wide plank being towed behind a launch. Charles Meade Ramus, rector of the village of Playden, Sussex, England, is credited with inventing the first feasible hydroplane form as early as 1870. Froude, in 1872, had tank tested a model of the hydroplane design at the request of the Admiralty but considered the concept to be unrealistic. Regardless, undeterred by the adverse opinions of the establishment, speed buffs doggedly awaited the development of engines of sufficient power to achieve their dream and eventually, in the first decade of the 20^{th} century, hydroplanes made their debut. A very early example, and possibly the first practical boat of that type, was a tiny hydroplane designed by a Frenchman, Claude Lelas. With an overall length of about 3 metres and powered by a single cylinder air-cooled internal combustion engine, the aptly named *Ricochet* reached speeds of more than 20 knots in calm water. Underneath, the hull was flat, like a punt, but to minimise the wetted surface area at speed and to achieve the correct trim for hydroplaning, the hull was also stepped, as Ramus had envisaged. For comparison with conventional designs, calculation of *Ricochet's* speed-length ratio was virtually meaningless, since so little of the craft was in the water when underway. Later models of *Ricochet*, fitted with more powerful engines as they became available, attained speeds in excess of 30 knots and by the end of the decade stepped hydroplanes had set the course for future high-speed powerboat design.

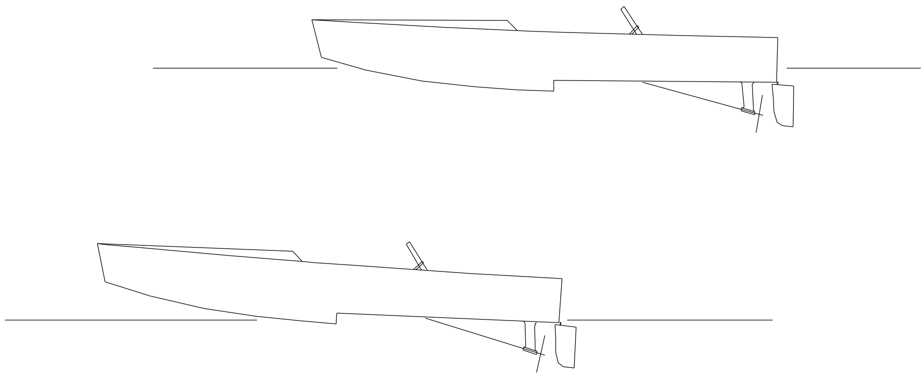


Figure 7-1

Figure 7-1 illustrates a small single-step hydroplane, at rest and at speed. The original idea of hydroplaning, or simply 'planing', was relatively straightforward, at least in concept, compared with the assortment of opinions pertaining to the design of vessels moving at relatively slow speeds such as ships and sailing yachts. Fundamentally, a planing hull was considered to be analagous to a rectangular plate held stationary within a jet of water trained directly at the plate's plane underside, as in Figure 7-2, hence the term 'planing'.

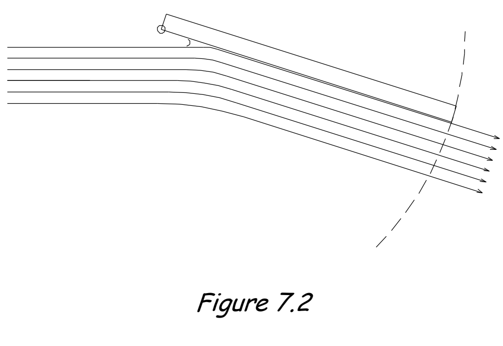


Figure 7.2

Shown hinged from its top, the plate hangs vertically in the absence of any flow of water but as the velocity of the stream is increased the plate tends to assume a more horizontal attitude. In equilibrium, the angle of the plate is primarily dependent on two factors, the weight of the plate and the velocity of the water. At no point is the plate actually floating. The forces supporting the plate are, instead, a direct result of the stream of water hitting the plate's underneath surface. Expressed in more scientific terms, at no stage is the plate supported by the forces of buoyancy but solely by the 'dynamic lift' brought about by the deflection of the water moving at high velocity. Ultimately, at very high stream velocities, the plate would be expected to skip horizontally on the surface of the water, giving an indication that, within reason, there may be no limit to the speed at which a hydroplane can travel. In 1978, little more than a century after Ramus's invention of the stepped hydroplane form, an Australian, Kenneth Peter Warby, on Blowering Dam in New South Wales, Australia, driving *Spirit of Australia*, a backyard-built, jet-powered hydroplane of his own design, achieved a world water-speed record of 300 knots.

World records aside, from the outset flat-bottomed hydroplanes quickly proved to be impractical in anything other than a complete calm. Froude's general assessment of the hydroplane form had been correct, it was unrealistic for everyday use. If speeds beyond the 'hull speed' barrier were ever going to be achieved efficiently by small powerboats under ordinary sea conditions, an alternative hull shape was essential, logically a coalescence of the long and slender traditionally shaped hull and the flat-bottomed hydroplane form. By the 1920s, the evolution of racing boats had led to the forerunner of today's fast powerboat designs, the 'runabout', intended for general use and brought to prominence in the USA by powerboat designers such as the renowned John Ludwig Hacker. Sporting the V-shaped forward sections of the traditional hull and the flat after sections of the hydroplane in a hard-chined stepless configuration, a typical Hacker-style 'runabout' of that era, shown in Figure 7-3, was very long by today's standards to maximise the waterline length, had a minimum beam to minimise the wave-making resistance and a transom stern, the bottom of which was dead flat to encourage 'dynamic lift'. An overall length of about 7 or 8 metres was generally considered to be about the minimum for adequate performance.

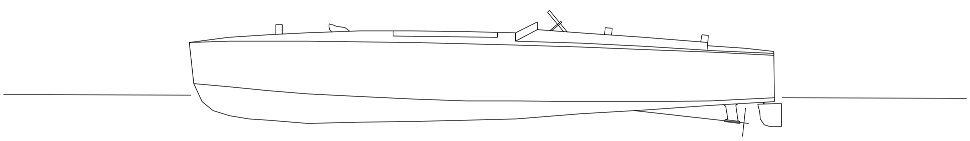


Figure 7-3

The distinguishing characteristic which sets planing hulls apart from non-planing hulls is the fact that at speeds above 'hull speed' the hull shaped for planing is bodily lifted above its 'at rest' position by the interaction between the hull and the water. As depicted in Figure 7-1, a planing hull at rest sits low in the water, buoyed up by the displacement of its own weight of the fluid. Moving forward slowly at speeds up to 'hull speed', the planing hull performs almost exactly as a traditionally shaped non-planing hull, creating a wake which is dependent on the vessel's speed and displacement, but much less efficiently due to a greater wetted surface area and a poorly streamlined shape. Major differences in performance between a planing hull and a traditionally shaped non-planing hull only begin to occur when each is pushed beyond 'hull speed'. Contrary to what might be expected, planing hulls do not display any evidence of the so-called 'dynamic lift' prior to reaching 'hull speed', which means that planing commences at different speeds for hulls of different waterline lengths, no better exemplified than in the case of a launch travelling at less than ‘hull speed’ towing a dinghy which is planing. To differentiate between non-planing and planing speeds, all speeds up to and including ‘hull speed’ are today logically referred to as ‘displacement’ speeds. Similarly, traditionally shaped non-planing hulls are generally referred to as ‘displacement’ hulls.

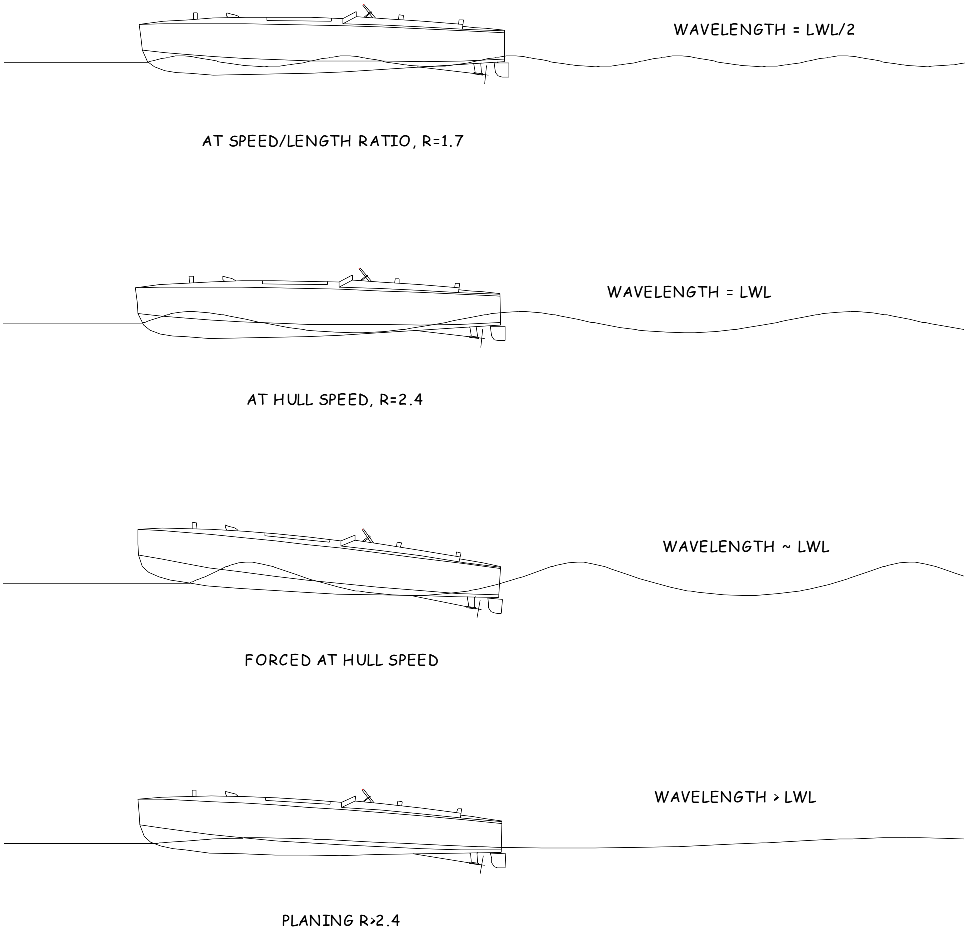


Figure 7-4

The stages through which a classic planing hull progresses as it accelerates to high speed are quite distinctive and worthy of examination. As shown in Figure 7.4, when forced beyond ‘hull speed’, the bow of a planing hull begins to rise and the stern drops, giving the impression that the boat is climbing its own bow wave and that the stern is sinking into the trough created by the first of the transverse waves within the wake. In that early stage of planing, the bow of the vessel might temporarily emerge from the water, shortening the hull’s effective waterline length and thereby increasing the wave-making resistance at that speed. During this transitional stage, between displacement speeds and true planing speeds, the wake and the resistance can become extreme, the noticeable ‘hump’ that so many sailors refer to after experiencing the sensation of a hull seemingly struggling to bodily haul itself up and over the bow wave. Pushed harder, the boat levels out, the bow drops and the stern begins to rise, giving a feeling to those onboard that the bow wave has finally been overtaken and the hull is suddenly lighter, riding high in the water and surfing the bow wave which, by then, has flattened, as has the wake in general. Aft, water streams cleanly from the transom stern, giving the appearance from onboard that the hull has an effective length that extends well beyond the transom. Mostly supported by 'dynamic lift', the vessel has, by that stage, successfully powered through the 'hull speed' barrier and is 'fully planing'. At higher speeds the planing hull is elevated even more above its 'at rest' position and the transom stern, the bottom of which always remains below still water level while planing, rises as the hull, like the flat plate in Figure 7-2, adjusts its angle of trim to match its speed over the water. The wake of a planing hull at high speed is minimal compared with its wake at 'hull speed' but still exists, mostly noticeable as a series of smaller diverging bow waves spreading at a decreased angle of incidence to the direction in which the hull is travelling.

Since the introduction of the 'runabout', fashionable in the 1920s, evolution of the planing hull has continued. Taking advantage of lighter and more powerful engines, powerboat designers have practically abandoned the concept of completely flattening the after sections of a planing hull, choosing instead to extend the V-shaped forward sections through to the transom stern, which, in a boat for general use, typically has a deadrise of about 15 degrees. However, in doing so, the lines of the planing area of the hull which come into contact with the water are generally kept straight longitudinally to preserve the flat plate effect. The result, illustrated in Figure 7-5, is the modern V-bottomed, hard-chined planing hull which experiences less pounding at speed in rough water than a flat-bottomed vessel but is generally considered to be somewhat less efficient in generating 'dynamic lift', hence the need for greater propulsive power.

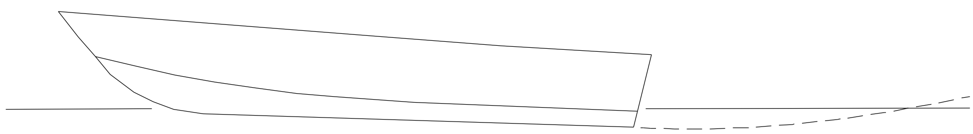


Figure 7-5

Remarkably, although Newton's theories regarding the resistance of an object moving through a fluid had long been abandoned for displacement hulls, the form resistance and 'dynamic lift' of a planing hull at speed were, from the outset, reasoned to be the result of the direct impact of the hull on the fluid particles located along its path, in line with Newton's original concept and completely logical if the analogy of the flat plate is accepted. In reality, the ultimate depth at which water particles are deflected by a planing hull is uncertain but, through extensive tank testing of flat plates, correction factors taking into account the length, beam, cross-sectional shape and angle of trim of the planing surface have been deduced so that by applying Newton's principles the form resistance and the lifting force of a powerboat hull's planing surface can be calculated with sufficient accuracy at the design stage.

Figure 7-6 illustrates the difference between the form resistance of a planing hull and that of a displacement hull at speeds beyond 'hull speed'. As anticipated by the early proponents of the hydroplane concept, the 'dynamic lift' generated by the flat areas of a planing hull raises the vessel above its 'at rest' position at speed with a significant reduction in the total underwater resistance compared with a displacement hull of equivalent length and weight. Above 'hull speed' the form resistance of a displacement hull escalates out of all proportion to any increase in speed, effectively creating the 'hull speed' barrier. For a planing hull, the form resistance increases above 'hull speed' but at a lower rate than that of a displacement hull. From Figure 7-6 it can be seen that beyond 'hull speed' and logically towards the end of the transitional stage between displacement speeds and 'fully planing', the rate of increase in the form resistance of a planing hull begins to diminish, the resistance curve reverses its steep upward trend and begins to climb steadily, indicating that given sufficient power a planing hull is capable of surmounting the 'hull speed' barrier and attaining higher speeds. In practice, the planing concept is particularly applicable to the design of small powerboats.

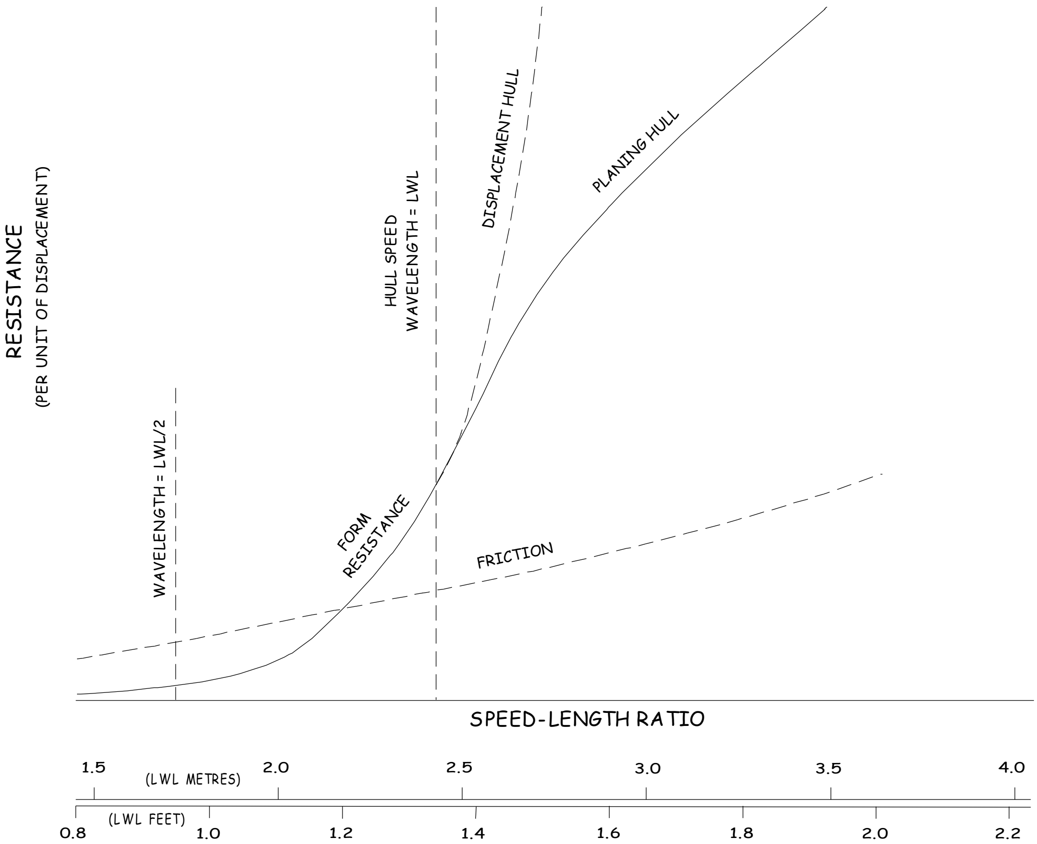


Figure 7-6

Under sail, the likelihood of a vessel achieving speeds much higher than 'hull speed' is not only dependent on the shape of the hull but, obviously, is also directly influenced by external factors such as the strength of the wind and a vessel's course relative to the wind direction. One consequence of having such a restricted source of power is that for speeds above 'hull speed' sailboats are encumbered with the added complication that hulls shaped for planing, no matter how similar they may seem in appearance and behaviour to the flat plate shown in Figure 7-2, are, in reality, not flat plates and are never, at any stage, solely supported by 'dynamic lift'. Even when supposedly 'fully planing', the wedge shaped portion of a planing hull below still water level, evident in Figure 7-5, continues to displace water, providing buoyancy and creating surface waves. Although empirical formulae to determine the lift and drag of planing hulls, based on the results of tank testing of flat plates, have proven adequate for powerboat design, those same formulae lack the precision required to perfect the shapes of relatively low-powered sailboats that, in many cases, are likely to exceed 'hull speed' only occasionally and by the smallest of margins, usually to gain a race-winning advantage.

Racing, not surprisingly, has been the impetus for many improvements in sailboat design since the formative years of sailing yacht racing as an international sport in the mid-19th century but, unlike powerboat racing, the goals of which are straightforward, sailboat racing did not evolve with the primary objective of achieving high speed on the water. Yachting, under sail and usually with a professional crew, emerged at a time when sail for commercial and naval use was already in obvious decline, unable to compete with steam. Prolonging, in some measure, the romance and tradition of the passing era, yachting was endorsed by the wealthy on both sides of the Atlantic as an unhurried diversion from the hustle and bustle of an increasingly industrialised world. Initially, sailing yachts were adaptations of traditional workboat designs, limited in their maximum speed by their inability to exceed the 'hull speed' barrier. To all intents and purposes, when large sailing yachts were later built specifically for racing, which was an inevitable eventuality for their naturally competitive owners, the very existence of the 'hull speed' barrier practically guaranteed close contests between sailing yachts of similar proportions. Deliberately, in due course, sailing yacht racing was formulated to be a contest of sailing skills, rather than an outright speed challenge. Rules discouraging 'extreme' designs were introduced to protect the status-quo and, over time, other rules and formulae were concocted, primarily based on vessels' waterline lengths, to allow sailing yachts of varying sizes to race against each other on a handicap basis. Nevertheless, by the time powerboat racing burst onto the scene in the first decade of the 20th century, the intensity of competition throughout the latter part of the 19th century had led to a spectacular evolution of the sailing yacht. Working within the natural limits imposed by the 'hull speed' barrier and the artificial constraints of the racing rules, sailing yacht designers, amateur and professional, had honed the traditional workboat hulls of the past into efficient, close-winded, sailing machines. Protection of the 'hull speed' barrier had, however, dictated the terms of development.

Far from the restrictive influences of the emerging yachting establishment, in the remote tropical islands of the Pacific, multihulls under sail had, of course, been effortlessly exceeding 'hull speed' for hundreds, if not thousands, of years. The long narrow hulls of the proas of Micronesia, and to a somewhat lesser extent the double canoes of Polynesia, were more like planks floating on their edge rather than on the flat, as in the hydroplane concept. Slicing through the water with very little tendency to make waves, Micronesian proas, in particular, were known to be capable of routinely achieving high speeds, sometimes sailing faster than the wind on a reach, probably to about twice their 'hull speed', a speed-length ratio of about R=5.0. Yet, although first encountered by Ferdinand Magellan's fleet early in the 16th century and subsequently praised by all Europeans who had personally witnessed their incredible sailing performance, eye-witness accounts of the speed of the Micronesian proa and, later, the less complicated Polynesian double canoe, were routinely dismissed in Europe as unlikely for craft deemed to be mere primitive sailing contraptions. Engulfed in their own maritime traditions and accustomed to monohulls exclusively, Europeans generally remained unresponsive to their discoveries of multihulls on the far side of the planet for more than four hundred years, until the mid 20th century. In the meantime, attempts to design and construct multihulls outside of Oceania were few and far between and, for the most part, were unsuccessful in advancing the multihull concept.

The first documented construction of a multihull in Europe was a double-hulled sailing vessel launched in 1662 at Dublin, Ireland, for an Englishman, William Petty. Born in 1623 into a poor family, Petty began his working life as a cabin boy on a merchant ship and, within two decades, had worked his way into the top ranks of society and accrued a small fortune. By all accounts, Petty was brilliant. Competent in Latin, Greek, mathematics and navigation by the age of 16, Petty advanced to being a professor of anatomy, a medical doctor, a high ranking government official and, at the time of his venture into multihull design, responsible for the ordnance survey and mapping of Ireland. Petty, by then, was also a foundation member of the council of the newly incorporated Royal Society of Natural Knowledge.

While living in Ireland, Petty, who regarded the sailing ship as the '*greatest and noblest of all machines',* designed and built four double-hulled vessels, each progressively larger than its predecessor and all intended as experimental. *Invention*, the first to be launched, displaced less than 2 tonnes, followed almost immediately by *Invention II,* weighing more than 30 tonnes and soon proven to be clearly capable of outsailing traditional vessels of similar size, not only in speed but also in the ability to sail closer to the wind and to more easily negotiate the shallow water of the local harbours. Before proceeding with the third design, Petty sailed his unorthodox craft back to England where the vessel was publicly ridiculed by one and all but, at the same time, marvelled at sufficiently by members of the Royal Society for Petty to gain support for his next project. *The Experiment* was subsequently launched at Dublin in 1664 with the king of England in attendance. As before, the new double-hulled vessel, about 18 metres in length, proved superior in performance to conventional designs but, after only a short period of trials, was lost at sea with all hands in a severe storm that also sank several other ships. In the eyes of the cynical public, Petty's bizarre experiments were immediately judged to be the utter failure which they had anticipated from the beginning. Dejected, but not defeated, it was not until 1682 that Petty launched his fourth and final double-hulled vessel, which performed abysmally, causing him to withdraw from society, resolving to devote the rest of his life to further analysis of his designs. Petty died three years later and, with him, any prospect of an immediate future for multihulls in Europe.

Long after Petty's experiments were abandoned, a spate of attempts by Westerners to utilise the multihull concept was prompted by the emergence of James Watt's steam engine in the late 18th century. Mostly steamships, along with a few riverboats that were actually horse powered, the common objective of those later ventures into multihull design was the spacing apart of two, sometimes three, hulls so that paddle wheels could be supported between them, allowing multiple paddle wheels to be installed while, at the same time, reducing the likelihood of the wheels being lifted out of the water by the rolling or pitching of the vessel or being damaged while docking, problems typically encountered by stern and side-wheelers. Some of those early multihulls truly were contraptions but, in contrast, two steamships built in the latter part of the 19th century for limited offshore use were, in their day, exceptional.

In 1874, *Castalia,* a double-hulled steamship of about 90 metres in length, was launched at London, England, for a British transport company to ferry passengers across the English Channel to and from France. With a beam of about 18 metres, *Castalia* was, at that time, the largest vessel to be employed in the Channel service, monohulls of similar size being excluded because of their deep draft which prevented them from entering some harbours at low tide. Although well received by a public that was more knowledgeable than in Petty's day and, by then, thoroughly accustomed to innovation, *Castalia* proved to be a commercial failure, primarily because the effort required to push the novel ship through the water had been underestimated. As a consequence of that error, the paddle wheeler had been fitted with engines that were extremely underpowered. Instead of being able to exceed 14 knots, as expected, *Castalia* could barely reach a top speed of 11 knots, at least 1 knot slower than the speed of its rivals, resulting in the ship being unable to meet the timetables of the connecting trains at either end of its regular run. *Castalia* was withdrawn from service in 1878, to be replaced by a more powerful and slightly larger double-hulled vessel, the paddle steamer *Calais-Douvres,* launched at Newcastle, England, the previous year. Capable of carrying as many as 750 passengers, the new ship was even more popular with the travelling public but, again, was disappointingly slow, achieving an operating speed of only 13 knots. As well, *Calais-Douvres* was very 'wet' in rough seas, necessitating the withdrawal of the ferry service during the more boisterous winter months but, despite those setbacks, the vessel managed to function somewhat successfully as a ferry for several years, conveying more than 50,000 passengers across the English Channel each year until 1887, when permanently replaced by a conventional steamship.

Castalia and *Calais-Douvres* were designed during the period when Froude's tank testing was in its development stage and, in hindsight, both ships were simply casualties of the lack of experience in multihull design in Europe in the 19th century. Despite their popularity, the two double-hulled steamships had not lived up to expectations, their downfall being speeds much lower than anticipated coupled with higher than average fuel consumption. Intended to operate at well below 'hull speed', in actual fact the faster of the two ships, *Calais-Douvres,* was incapable of exceeding a speed-length ratio of about R=1.4, a speed at which surface friction, not wave-making, is the principal resistance to forward motion, as illustrated in Figure 5-5. From the outset it was inevitable that, with a sizeable wetted surface area because of an extra hull, each of the two double-hulled vessels would be subjected to greater frictional resistance than monohulls of similar length and displacement. Not until speeds approaching 'hull speed', when wave-making becomes the dominant cause of resistance, does the multihull with its slim lines begin to gain a distinct speed advantage over a monohull, a fact of which many yachtsmen in the USA already had first-hand experience, mostly to their displeasure.

Less than two years after the unveiling of *Castalia*, in 1876, another revolutionary 'double boat', diminutive by comparison, had been quietly launched into the waters of Bristol, Rhode Island, USA. The brainchild of a 28 year old graduate mechanical engineer, Nathanael Greene Herreshoff, the unusual craft, *Amaryllis,* essentially consisted of two identical and extremely narrow hulls, each having a length of slightly more than 7.5 metres and a beam of only 0.5 metres. Spaced apart at about 4 metre centres, the two hulls were joined with an open bridgedeck to which a single rudder was attached on the centreline. A sailboat, built primarily as an experiment, *Amaryllis* initially sported an unusual rig which utilised a mast stepped in each hull but, after a brief trial, the novel craft was converted to a sloop rig typical of the small racing yachts of that era, with a large gaff main and a small jib. During the trials *Amaryllis* was estimated to have reached speeds of about 18 knots in bursts, a speed-length ratio in excess of R=6.0 and almost three times the vessel's 'hull speed' of about 6.5 knots. For its length, *Amaryllis* was rightly reckoned to be the fastest vessel in the world at that time, whether powered by sail or steam.

Herreshoff, appreciating the impact his creation would likely have on sailing yacht racing, without more ado sailed *Amaryllis* down the coast to New York to participate in the prestigious Centennial Cup regatta, creating a dilemma for the race organisers. Dubbed a 'catamaran' by the New Yorkers, who weren't quite sure what to make of such an ungainly vessel, *Amaryllis* was courteously granted unanimous approval to compete, mainly because most thought the uninspiring little 'life-raft' type of vessel would be unable to complete the course, let alone win. But, to everyone's surprise, after a slow start in a light breeze the wind eventually strengthened and *Amaryllis* scurried past the entire fleet as if the other competing yachts were anchored, comfortably taking line honours to the applause of thousands of spectators who had witnessed the catamaran's incredible performance. Inevitably, a protest was lodged and, as a consequence, multihulls were banned from future competition. The general consensus was that the owners of large schooners simply did not want to risk having their vessels instantly converted into 'useless lumber', as one observer commented, but, in fairness to the yachting establishment, the selection of a particular type of vessel to suit their specific purpose was justifiable. Certainly, there are genuine reasons why a ballasted monohull, no matter how slow, might be judged to be the preferred choice of vessel for racing, not the least being the ability to recover from a knockdown. Interestingly, Herreshoff, in tacit support of the establishment's decision, personally expressed the view that catamarans were ideally suited to daysailing in protected waters and, in fact, several were subsequently designed and built for that purpose. Rather than pursue the multihull concept, though, Herreshoff chose instead to be a designer of conventional yachts and, incredibly, after their sensational debut, catamarans faded completely from the racing scene and would not reappear for almost seven decades.

Although the opportunity to understand and develop the multihull concept to overcome the 'hull speed' barrier had been rejected by the yachting establishment, genuine advances in sailboat design continued to be made elsewhere. In those last few years of the 1800s, beyond the guarded realm of keelboat racing and unhindered by the complexity of measurement rules intended to equate the performances of expensive sailing yachts of varying sizes and types, sailboat racing emerged throughout the Western world as a popular recreational pursuit for the less affluent working classes. Typically, competition began with jousts between local inshore fishing vessels or the like but soon evolved into regular contests between highly specialised craft built solely for racing. In the main, the boats were relatively small and inexpensive which allowed and encouraged the boats' owners and builders to experiment and to test their ideas directly on the water in competition. Almost unwittingly, recreational sailing in small boats became a test bed for innovation in hull design, generally more effectively than the inevitable use of tank testing for the design of expensive racing yachts. Predictably, some boat types were destined to become truly 'extreme', pushing the boundaries of design, construction and sailing techniques well beyond ordinary experience, all for the sake of speed. The world famous Sydney Harbour 18-foot Skiff, raced continuously as a highly competitive development class in Australia since that era, is a prime example of the type. In many ways, the evolution of the '18-footer' is a reflection of the countless changes that have taken place in recreational sailing since the late 1800s to the present-day.

18-foot Skiff racing began on Sydney Harbour in 1892 with the intention of turning sailing into a spectacle that could be enjoyed by crowds of enthusiasts lining the harbour's foreshores. The rules were simple, boats were to have a length of 18 feet, a fraction under 5.5 metres, sail area was unlimited and the number of crew optional. Developed from the clinker rowing skiffs that once worked the harbour, the first '18-footers' were open, centreboard sloops of either clinker or carvel construction with steam bent frames. Gaff-rigged, the masts were of solid timber and the sails were cotton. As could be expected, sail areas were huge, often totalling more than 300 square metres including mainsail, ringtail, topsail, ballooner, water sail and spinnaker. Bowsprits extended the sail plan beyond the bow by more than a boat length while booms, at almost twice the length of the hulls, protruded a similar distance aft. Crew numbers were dependent on the wind strength but usually ranged between 10 to 15, including a boy to bail. Hull designs varied and, in those early years, were of conventional displacement shapes, having a beam of about 2.4 metres for stability but with an emphasis on narrow waterlines. From the outset, speeds above 'hull speed' were common off the wind and, through a process of trial and error, over the first few decades of competition hulls gradually evolved towards planing shapes with flattened sections aft.

Uffa Fox, a boatbuilder and designer in Britain, is credited with being the first to design a dinghy hull which permitted planing under sail, in 1928. Clearly, there were sailboats elsewhere that had already acquired the ability to plane to some extent but Fox's publicised ideas on hull shapes, inspired by the fast powerboats of the 1920s, had an impact internationally, causing a revolution in sailing dinghy design. As part of that revolution, in the early 1930s a radical new '18-footer', *Aberdare*, was launched on the Brisbane River in Queensland, lighter and narrower than existing boats, with a reduced sail area and crewed by as few as 7 or 8 men. Hugely successful against the bigger Sydney boats, the skiff was reliably clocked over a measured mile at 23 knots, a speed-length ratio of R=9.8, four times the boat's hull speed of 5.7 knots. The success of the new design concept marked the beginning of the end of the 'big boat' era in 18-foot Skiffs and started a trend towards even smaller boats with fewer crew.

Another revolution in '18-footer' design began about two decades later, in the 1950s, with the availability of reliable waterproof glues and plywood suitable for marine use, bringing to an end the heavier traditional clinker and carvel construction methods of the past. Lighter hulls fitted with smaller rigs that took advantage of synthetic sails, glued hollow spars and trapezes, required less crew weight to keep the boats upright and crew numbers quickly dropped to 6, then 5, then 4. Ultimately, in 1959, one young designer-builder, Bob Miller, launched *Taipan*, an ultra-lightweight 3-man boat that was capable of planing easily in medium breezes, practically making all previous skiff designs obsolete. Incidentally, to gain every advantage *Taipan* also sported horizontal wings on its rudder for improved efficiency and the original centerboard was fitted with a winged endplate. Miller, some 24 years later, would become famous throughout the world as Ben Lexcen, the designer of the winged-keel *Australia II,* the yacht which ended, in 1983, the USA's 132 year stranglehold on international yachting's premier trophy, the America's Cup.

During the 1950s and 1960s, the advances in design and construction that transformed 18-foot Skiff racing also spread rapidly throughout Australia's other skiff and development classes. Many of the innovations originated in New Zealand, where skiffs and other classes common to Australia were sailed, but, despite the dynamic sailing scene 'down under', those innovations in design and construction largely went unnoticed in the northern hemisphere. In Britain, high speed sailing headed in a different direction with the return of catamarans to the racing scene, a trend which steadily gained momentum worldwide. For 18-foot Skiff racing in Australia, however, it was from the design of a simple, family-oriented sailboat, the NS14, that a crucial breakthrough in planing hull shape would evolve.

Originially known as the Northbridge Senior, the NS14 was conceived in 1960 by a group of senior members of Northbridge Sailing Club on Sydney Harbour as a simple development class that would not demand abnormal strength or acrobatics from its crew. Intended as a boat suitable not only for senior sailors but also for family crew combinations, the NS14 was an instant success. Based on an existing New Zealand hull design, at 14 feet in length, about 4.27 metres, and carrying a maximum sail area of only 100 square feet, about 9.3 square metres, without a spinnaker or trapeze, the NS14 was considered tame by most Australian sailors. Yet, the class quickly grew and racing became intense, attracting some of the best sailing talent, many of whom were lured by the opportunity to implement their individual thoughts on design without the huge financial outlay typical of many development classes, particularly keelboats. The result was a rapid refinement of NS14 rigs and hulls to make efficient use of the very limited driving force that was available. Flexible over-rotating timber masts soon became popular within the class to improve the efficiency of the small rigs but it was in the reduction of the resistance of the hulls at marginal planing speeds where huge gains in performance were made. By the late 1960s, less than a decade after their introduction, NS14s were beginning to acheive planing speeds off the wind with unprecedented ease in moderate conditions, without experiencing the 'hump' in resistance that, previously, was universally accepted as being an inevitable feature of the planing process. Despite their small sail area and, as a consequence, their low power to weight ratio, for NS14s the 'hull speed' barrier had become almost non-existent.

The rapid refinement of the NS14 was evolutionary and a result of input from many quarters, but the undisputed authority on NS14 design and performance in those early years was a co-founder of the class, Frank Bethwaite, a New Zealander living in Sydney. An aircraft pilot by profession, throughout the 1960s Bethwaite used his extensive practical knowledge of aerodynamics to experiment with NS14 rigs to help devise techniques to extract the maximum power from the boats' small sails, all the while measuring, recording and regularly publishing his findings. As well, Bethwaite methodically investigated the subtle differences in the shapes of successful NS14 hulls, resorting to a simple but practical method reminiscent of Froude's experiments when initially exploring the matter of a hull's resistance a century before. Makeshift outriggers were fitted to a small powerboat so that an NS14 on each side could be towed around the harbour, allowing the resistance of a hull to be measured and compared with a constant test rig over a range of speeds for various displacements and angles of trim in a variety of sea conditions. The data collected by Bethwaite showed significant variations in resistance for hulls that appeared only marginally different in shape, eventually leading to further refinement of what he would later refer to as the 'dynamically humpless hull', which could exhibit a smooth resistance curve as speed exceeded 'hull speed'.

From the 1970s onwards, Bethwaite's expertise began to infiltrate 18-foot Skiff design. At that time, the fastest NS14s had very low rocker, fine bows with straight narrow waterlines leading to almost parallel chines and flattened cross-sections aft. Low planing speeds were achieved without the forward sections emerging from the water. The '18-footers' developed those trends further to take advantage of their huge sail areas and their hulls became wider, especially aft, creating almost a wedge shape in plan view. Over the next few decades, again with input from many quarters, the performance of the '18-footers' improved remarkably, facilitated not only by improvements in hull design but also by revolutionary developments in construction techniques, making use of materials such as fibreglass, Kevlar and carbon fibre to produce much lighter and stiffer hulls. Aloft, aluminium, which had already replaced timber as the material of choice for masts and spars was, in turn, replaced by stronger and lighter carbon fibre. To give crews greater leverage, wings were added to the hulls, encouraging the use of bigger sails that were also made from lighter and less porous plastic materials. Bethwaite, always in the background, assessing designs and publicising his thoughts, continued to influence 18-foot Skiff performance into the early years of the 21st century by which time the '18-footers' were capable of planing unpwind in true wind speeds as low as 8 knots and, off the wind, could achieve speeds of double the true wind speed, necessitating a complete re-evaluation of traditional sailing techniques.

The progress that has occurred in the design, construction and handling of 18-foot Skiffs from the time of their first appearance on Sydney Harbour in the late 1800s has, without doubt, been extraordinary, yet the '18-footers' still do not represent the ultimate in sailing performance. Since the return of catamarans to small-boat racing in the 1950s, multihulled sailboats have not only become increasingly popular but also extremely fast on all points of sail, so much so that multihulls have now replaced monohulls as the vessels of choice in which to compete for the prestigious America's Cup. Incredibly, although multihulls under sail have been evolving for hundreds, if not thousands, of years and have long been capable of reaching very high speeds, the modern multihull, under sail or power, is still in the early stages of development. Even for experienced multihull designers, the reasons why multihulls are able to negotiate the 'hull speed' barrier with such apparent ease still remain somewhat blurred. In fact, despite several centuries of scientific investigation, the theory of the wave-making process of any hull, at any speed, whether it be a multihulled America's Cup competitor, an 18-foot Skiff or the largest ocean-going ship, has never been satisfactorily resolved.

After Froude, a critical step forward in the mathematical determination of wave-making resistance seems to have been taken when Thomson made the simple but drastic assumption in 1887 that the entire moving hull could be collapsed into a single point. Sailing yacht designers of that era had, with relative success, already embraced Archer's *Waveform Theory* in lieu of any logical and usable alternative but, for academics, such a simple hypothesis would never suffice. Not only did Archer's theory ignore the complications of a real fluid, problems such as a boundary layer which gets dragged along with the hull or eddies which might be formed in the wake, fundamentally the world of sail and the world of commercial and naval shipping, the main concern of the academics, were at odds. Archer's aim had been to provide a simple theory by which sailboat designers could more accurately shape a hull of known waterline length and displacement to minimise wave-making resistance generally. Academics involved in commercial and naval ship design projects were, on the other hand, primarily concerned with determining the actual wave-making resistance of arbitrarily shaped hulls intended for operation within a narrow range of speeds, to ascertain both the powering requirements and the performance characteristics. By the end of the 19th century, designers of all types of vessels had come to the stark realisation that because of the complexity of the issues involved, there was never going to be a simple solution to the question of a vessel's wave-making resistance. Archer's *Waveform Theory* was proving to be inadequate and further investigation of the wave-making process was slipping, once again, into the hands of elite mathematicians. Designers, it seemed, were destined to rely, for ever more, on tidbits of almost unintelligible information gleaned from academia and, until that division could be bridged, evolution, not science, would continue to play the lead role in the development of sailboats.

Throughout the comparatively short history of recreational sailing it has often been designers without formal qualifications who have been at the forefront of development, reinforcing the belief that the principles of design are still very much open to intuitive interpretation and experimentation. Australia's Ben Lexcen, designer of the wing-keeled *Australia II* that ended the USA's dominance of the America's Cup in 1983, had a mere three years of schooling, beginning at the age of eleven. As a young man, at the start of his design career, Lexcen had revolutionised Sydney Harbour 18-foot skiffs and went on to create several internationally successful ocean racing yachts before tackling the International 12-Metre class for the 1974 America's Cup challenge. Ultimately, by utilising his depth of natural talent and experience Lexcen was able to surpass design teams of formally qualified naval architects to capture the world's premier yachting trophy. Lexcen was able to achieve the pinnacle of sailboat design not only because he was a creative genius who lived and breathed boats but, presumably, also because he was not constrained by a formal education, qualities possessed, no doubt, by Uffa Fox and many other outstanding sailboat designers, including, from the USA, one of Lexcen's foremost adversaries during his design career, Olin Stephens. Described in his lifetime as the doyen of the world's yacht designers, Stephens was older and more experienced than Lexcen, his extraordinary career spanning more than half a century from his first design in 1928 until his retirement in 1982, during which time he was responsible for more than two thousand designs, mostly ocean racing and cruising yachts and six successful defenders of the America's Cup. Stephen's designs not only paralleled the development of the modern ocean racing and cruising yacht but, through refinement and innovation, were influential in their evolution. Stephens, like Lexcen, was primarily self-educated, having completed only a few months of formal engineering training. Even today, in the 21st century, although many, but not all, sailboat designers are formally qualified as naval architects, the most successful remain largely 'self-taught' in their preferred area of expertise.

Modern sailboat design is often presented as 'state-of-the-art' technology, a notion reinforced, no doubt, by the ever-increasing sophistication of hull design software that has proliferated since the introduction of personal computers into design offices during the 1980s. In reality, though, while improved computer capabilities have helped designers to access the knowledge of academia, science is yet to provide complete or even satisfactory answers to some of the most fundamental questions of design, particularly those relating to the resistance caused by the formation of waves. For the creation of fast and efficient sailboats much of the theory, particularly that relating to the shaping of the hull, remains imprecise, offering only broad principles rather than the universal application of uncompromising mathematical formulae. The process that has led to the design of vessels capable of breaking through the 'hull speed' barrier to attain high speeds under sail, has been mostly evolutionary. For multihulls, the 'hull speed' barrier was smashed long before recorded history within Oceania, possibly without the existence of a so-called 'barrier' ever being recognised. For monohulls, it was not until the 20th century that the 'hull speed' barrier was finally broken convincingly, leaving in its wake an even greater void in the understanding of the wave-making process, the stumbling block which has persistently prevented theorists from achieving perhaps their greatest goal of all, the mathematical determination of hull shapes of least resistance.

A COMPONENT WAVEFORM THEORY

THE COMPONENT WAVE

Asked to sketch a modern sailboat, for the hull it is likely that a complete novice would draw a sleek profile with a raked bow, a straight sheer, a reversed transom and a low streamlined deck. To achieve a longer waterline length a sailor, more experienced, might choose to draw, instead, a vertical bow profile, a feature that the novice would undoubtedly consider to be 'slow' and unfashionable. Urged to define the underwater shape of the hull, or hulls, each would probably opt for sharp forward sections to cut through the water cleanly with a minimum of fuss, easing out gradually towards the mid-section and creating a slightly concave appearance near the bow at the waterline. But then what? Beyond those rough ideas, the design process becomes more than just a matter of aesthetics and vague perceptions. For a novice with some understanding of mathematics, a plausible suggestion for the waterline in plan view might well be a variation of a sine curve, which would result in a steady and precise change of direction along the hull to encourage a smooth flow of water, terminating in an identical concave shape aft. But what of the flow beneath the surface? Is it not equally reasonable that along the centreline of the hull the water should be encouraged to follow a similar curve? To apply the same logic to all of the imagined 'streamlines' would inevitably result in a bulging underwater shape, squeezed from all directions into a hollowed form at each end. As sensible as that concept may seem at first, on the water, after thousands of years of development, no such hull shape has evolved.

John Scott Russell, inventor of the *Waveline Theory*, appears to have had little or no experience in boating when commissioned to evaluate hull shapes for canal boats intended to operate at higher than usual speeds. So much so, Russell once admitted that after conceiving the idea for the 'waveline' bow, he had rushed to the nearest harbour to examine the boats there, only to find that many were shaped that way already. It is not difficult to imagine the twists and turns that Russell must have experienced in trying to adapt his theory to match the after sections of the vessels' hulls. Colin Archer, on the other hand, was much more experienced than Russell when he put forward his *Waveform Theory*, having already built and sailed boats to his own designs. As well, Archer had the advantage of being able to study not only the works of Russell, who had broken new ground alone, but also the creations of other successful sailboat designers and the results of William Froude's investigations into resistance. Today, all of that information, and much more, is readily available to the novice, yet step by step guidance on how to design a hull without reverting to evolutionary methods remains practically non-existent. Typically, the modern design process begins with a known hull shape which is then modified to the designer's liking. Numerous mathematical formulae are applied to determine some of the modified hull's characteristics, after which the results obtained from the various calculations are compared with those for similar hulls known to be successful under the conditions for which the new design is intended to operate. Through a cycle of trial and error, the designer eventually arrives at what is hoped to be an improved hull shape. Computers, obviously, hasten that evolutionary design process but, to date, the mathematical determination of an optimum hull form from scratch remains a fantasy.

The expectation of being able to achieve a 'perfect' hull shape using mathematical methods alone was instigated by the scientist Isaac Newton in the mid-1600s when he first suggested the likelihood of a uniquely shaped 'solid of least resistance' for a fully immersed object moving through a fluid. For an object moving, instead, across a fluid surface, the most efficient shape had, of course, already been a source of much deliberation by sailboat designers for thousands of years but, after Newton, the elusiveness of attaining such a goal has since enticed influential input from assorted intellectuals, some with very little practical knowledge of boat behaviour. Newton's logic was uncomplicated, basically assuming that an object's forebody simply collides with the fluid particles in its path, causing a resistance to forward motion. In the 1700s, Leonhard Euler, a brilliant mathematician, steered the scientific investigation of the form resistance of surface vessels away from Newton's simple logic towards the use of streamlines. Equally inexperienced on the water, John Scott Russell, an engineer, formed his radical ideas on hull design in the mid-1800s while making observations from the shore. On the other hand, William Froude, another engineer from Russell's era but with wider boating experience and well aware of the complexity of the task, favoured evolutionary methods to achieve improvements in hull design, practically avoiding the theoretical approach altogether yet was championed by many eminent academics. Remarkably, though, despite all of the scholarly input since Newton, it has been the uncomplicated theory proposed by Archer, a largely self-taught sailor, designer and builder of sailboats, that seems to have come closest to achieving, in a practical sense, a mathematical approach for the determination of hull shapes of least resistance.

Archer, in his less sophisticated search for design perfection, almost certainly tussled with various methods to refine the shape of a hull. For some unknown reason, a natural tendency of sailors is to visualise streamlines weaving around the hull as if the water surface remains undisturbed. At some point, Archer may have considered whether the flow of water around the hull could be dealt with most effectively by employing a precise distribution of the wetted surface, encouraging the streamlines to spread themselves uniformly at each cross-section. Such a concept had already been suggested and, like the need for smooth hull lines, an ordered distribution of the wetted surface is, surely, a contributing factor in the design of an efficient, easily driven hull form. For Archer, though, it had become evident from his knowledge of existing designs that although smooth lines and an evenly distributed wetted surface were important, alone they do not guarantee a hull shape that can move through the water with a minimum of resistance. In due course, Archer settled on an all-encompassing approach of utilising a precisely calculated distribution of the hull volume, reasoning that when combined with smooth hull lines by an experienced designer a more efficient hull shape could result. Archer, like Russell, endeavoured to compensate for the creation of the inevitable surface waves within the wake by visualising the hull moving past a fixed point, pushing a volume of water aside with its forward sections in a prescribed manner while the after sections are arranged to complement, perfectly, the natural return of the fluid into the imagined cavity created by the passage of the hull's largest cross-section.

Adapting Russell's ideas to his own, Archer's specific objective in structuring his *Waveform Theory* was to minimise the form resistance of a hull as it moved across the water surface at speeds approaching 'hull speed'. At the time, the entire maritime world was uncertain of the future of planing as a practical concept, particularly for sailboats, and consequently, for Archer, the need to widen the scope of his ideas to beyond 'hull speed' simply did not exist. Archer's practical approach to determining the optimum distribution of a hull's underwater volume was initially welcomed with open arms by sailboat designers desperate for mathematical support but, over time, the *Waveform Theory* proved to be only moderately successful for its intended purpose. Interestingly, though, a 'curve of areas', undefined but reminiscent of Archer's 'waveform', has lingered on as a crucial design criterion for all types of surface vessels, at all speeds. Surprisingly, a precise explanation as to why Archer's seemingly logical approach ultimately failed was not forthcoming from the scientific community. Instead, in lieu of pursuing Archer's rationale, academia eventually opted to support the extremely simple explanation, of how the wake is formed, by the scientist William Thompson but scientific grounds for the undeniable relationship between the distribution of a hull's displacement and its performance at all speeds, including planing, have since remained noticeably vague. In hindsight, Archer's straightforward approach to minimising a hull's resistance to forward motion was not far removed from that of Newton's for a fully immersed object and, given the continuing elusiveness of each of their goals, the prospect emerges that by casting aside Newton's simple logic in the 18th century, academia may have inadvertently over-complicated the analysis of a hull's wave-making resistance and set future investigation a more tortuous route.

For the novice investigating the complexities of hull performance, there could be no simpler or more direct first approach, surely, than personally experiencing the actual forces involved in propelling a boat by hand. Probing the performance characteristics of a small, traditional rowing dinghy, for example, can be most enlightening. After mastering the use of the oars, the novice soon realises how little effort is required in calm water to propel the boat at 'cruising' speeds which, in more technical terms, is usually at a speed-length ratio of about R=1.7. At that speed the waves within the wake seem almost irrelevant and the dinghy feels light, as if gliding across the water surface. Often to a novice rower's amazement, if the oars are suddenly removed from the water at 'cruising' speed the boat does not immediately stop, as might be expected, but slows very gradually, continuing on its course for some distance. Rowing the same boat at higher speeds, though, is markedly different. Although the effects of wind resistance and surface friction on the boat's performance may be anticipated by anyone, the consequences of altering the speed of a rowing dinghy, or even its trim or displacement, are not as straightforward. As speed is increased, the effect of the wake becomes more evident to the rower and the extra effort required to move the boat makes the likelihood of reaching 'hull speed' seem very remote. Unexpectedly, if the oars are suddenly removed from the water at those higher speeds the boat does immediately slow down, initially to about 'cruising' speed, after which it then continues for some distance as before. If an observer is taken aboard to assist with further trials, positioned aft, the dinghy trims by the stern and the rowing becomes strenuous, out of all proportion to the added weight. Achieving the previous 'cruising' speed under that circumstance might well be impossible but, unexpectedly, if a second passenger is added forward so that the boat resumes its designed trim, the effort required to row lessens, despite the increased load. Within each of those observations lie clues to the mystery of a hull's wave-making resistance. Somewhat illogically, however, it is not while rowing but when resting on the oars to mull over the whole experience that perhaps the most valuable clue to solving the mystery of a hull's wave-making resistance becomes evident. As momentum inches the boat slowly forward, the drops of water falling from the tips of the oars onto the water surface below do not create the familiar circular patterns of raindrops but, instead, miniature wakes, exactly as portrayed by Thomson, who was evidently not only an extraordinary scientist but also a perceptive yachtsman. By Thomson's own admission, his inspiration for determining the principles of how the wake is formed came to him while casually observing the wake created by a fishing line trailing alongside.

Any object propelled across the water surface creates waves, forming the characterisitic wake. Impossible to observe directly within the wake, however, are Thomson's hypothetical circular wave patterns that he maintained are created by the forward motion of a hull at every point along its course. Clearly evidenced by the drops of water falling from the tip of an oar to form a 'Kelvin wedge' on the water surface below, Thomson's circular wave patterns, despite their invisibility, do actually exist, combining immediately to form the wake described by Froude, a series of diverging crests coming off the bow and a series of transverse waves keeping pace astern. Amazingly, because they can never be observed individually, the very existence of the circular wave patterns within the visible wake remained unsuspected by scientists until Thomson's discovery in 1887, even though for centuries water waves emanating from a point source of disturbance had been commonly used to demonstrate the propagation of sound waves emitted from a fixed point and, as far back as 1842, an Austrian scientist, Christian Doppler, had extended the scientific knowledge of sound waves to include the effects of a moving source. Russell, working briefly as an engineer on steam-powered vehicles before moving on to investigate the wave-making resistance of canal boats, had actually been one of the first to carry out informal field experiments on the phenomenon later explained by Doppler's theory but apparently did not appreciate that Doppler's explanation was equally applicable to the waves created by a disturbance moving across the surface of a fluid. Had Russell made that connection, his ensuing research may have led to a more accurate determination of a hull form of least resistance than did his *Waveline Theory* but it is also likely any such breakthrough by Russell at that time may have been disregarded by the upper echelon of the scientific community, as was his discovery of the soliton, his 'Great Wave of Translation' which, throughout his lifetime, Russell insisted was of fundamental importance, to no avail.

Today, most people are at least vaguely familiar with the so-called Doppler effect, the change that occurs in the perceived wavelengths and frequencies of waves propagated from a source of sound which is moving relative to the observer. In Doppler's era, few occurrences of the phenomenon were observable, the most evident being the change in pitch of the locomotive's whistle as a steam train passed by. Today, a typical example is more likely to be that of a siren on a passing emergency vehicle. In each case, the pitch of the sound heard by a stationary observer varies from high to low as the source of sound passes. The change in wavelengths and frequencies of the waves reaching the observer are dependent on the observer's location but despite the observer's perceptions, it is important to realise, however, that the wavelength and frequency of the waves being emitted from the moving source actually remain constant throughout.

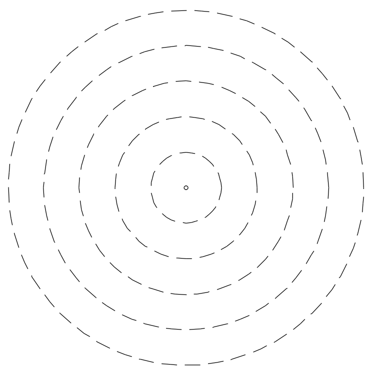


Figure 8-1

Figure 8-1 illustrates a wave pattern propagated through a medium by a stationary source of constant sound. Unlike waves on the surface of a fluid, sound waves are, in fact, longitudinal waves in which the particles of the medium vibrate back and forth along the line of the waves' travel, creating a series of compressions and rarefactions within the medium. Not too dissimilar, however, from the two-dimensional wave pattern created by a raindrop falling into a puddle, the compressions and rarefactions created by a sound spread out equally in all directions, resulting in a pattern of waves expanding outwards in three dimensions. Using the analogy of the raindrop, the compressions caused by sound can be represented by the crests of the waves on the surface of the water and the rarefactions by the troughs. As for the surface waves created by the raindrop, the amount of energy being delivered to any one place by sound waves naturally decreases as the wave pattern expands and, consequently, as the wavefront radiates outward through the medium the amplitudes of the oscillations become smaller and smaller until eventually, at some point, the sound fades completely. Doppler's discovery, the effect on the wave pattern caused by a moving source of sound is illustrated in Figure 8-2. Although the wavelength and frequency of the waves being emitted from the source remain unchanged from Figure 8-1, moving the source of sound, in this case at a constant speed from right to left, has the effect of shortening the wavelengths ahead of the source and lengthening those behind. The frequencies of the waves change accordingly, affecting the pitch of the sound heard by a stationary observer located anywhere within hearing range.

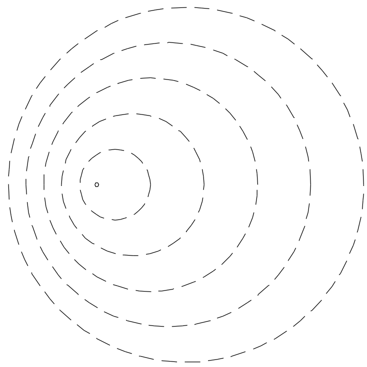


Figure 8-2

If the source of the sound increases its speed to that approaching the velocity at which the wavefronts are expanding, in other words to the natural velocity of sound for the medium through which it is travelling, the 'crests' of all the waves emitted by the source at each point combine ahead of the moving source to reinforce each other, increasing their amplitude, or loudness, enormously. Interestingly, for that phenomenon to occur, the source of the disturbance does not necessarily have to be an actual sound. Any object travelling through a medium, whether a source of sound or not, apparently disturbs the particles within that medium in a similar manner, creating the 'corpuscular waves' observed by Russell. If the object increases its speed until it reaches the natural velocity of sound waves through that medium, the waves caused by the disturbance combine to create the so-called 'sound barrier' at which point the frequency and amplitude of the vibrations can be so severe as to cause the object to disintegrate. Figure 8-3 illustrates the effects of a disturbance moving at the velocity of sound.

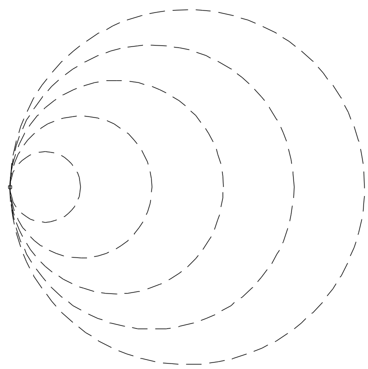


Figure 8-3

At speeds higher than the natural velocity of sound, the source of the disturbance travels through the medium faster than the pattern of waves expanding outwards from each point in its path, leaving the waves to combine in the object's wake as illustrated in Figure 8-4. Apart from being three-dimensional and enclosed within a cone rather than the two-dimensional V-shaped 'Kelvin wedge', the wake of an object moving through a medium at supersonic speed is not too dissimilar from that of the wake of an object traversing the surface of a fluid, as portrayed by Thomson.

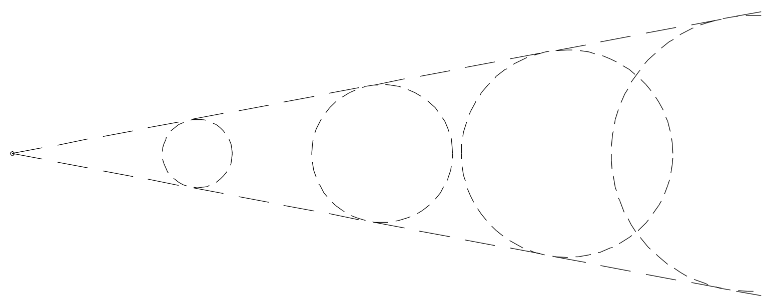


Figure 8-4

From a sailor's perspective, assuming Thomson's interpretation of the wake to be correct, an obvious question that immediately comes to mind is why, then, do surface vessels always appear to be moving at 'supersonic' speeds? Even when ghosting, the waves within a sailboat's wake, apart from a few tiny ripples near the bow, are never projected forward, as were the sound waves depicted in Figure 8-2, but are always contained within the characteristic wedge-shaped pattern. Without too much deliberation, there are at least two plausible explanations for that phenomenon. Firstly, unlike waves on the water surface, sound waves travelling through a medium do so at a constant velocity, regardless of their wavelengths, an observable fact which enables, for example, listeners to hear in proper sequence a melody produced some distance away by a variety of musical instruments. As well, the velocity of sound waves is relatively high, allowing sound waves to spread far ahead of a slower moving source such as a locomotive. In contrast, most waves on the surface of a fluid travel at speeds which are directly dependent on their wavelengths. Compressing the surface waves that might be expected to form ahead of a surface vessel, as in Figure 8-2, could possibly reduce the velocity of those waves to such an extent that, for a vessel at any speed, the component waves are immediately overtaken by the vessel and lost within the wake, never to be observed individually. Alternatively, an even simpler explanation of the phenomenon could be that the velocity at which the component wave pattern spreads on the surface of the water might always be slower than the speed of the vessel creating the waves. In any case, the visible wake of a surface vessel is what it is and, for the sailboat designer, a more pertinent question is how knowledge of the unobservable component waves can facilitate the shaping of a hull that could be expected to experience a minimum of wave-making resistance.

Much as Newton suggested for an immersed object, the wave-making resistance of a surface vessel at any particular instant is the resultant horizontal component of all the forces being applied to the hull by the surrounding water particles as the vessel attempts to move forward, pushing those water particles aside. Before Newton, from the science of hydrostatics, there was already an awareness that even a stationary vessel floating on the surface of the water experiences forces applied to the hull by the surrounding water particles but the resultant horizontal component of those forces acting on the stationary hull is, in fact, zero and the resultant vertical component of those same forces is, as Archimedes discovered, equal to the weight of the vessel. From the later science of hydrodynamics came the realisation that once there is relative motion between the hull and the water, the forces applied to the hull by the surrounding water particles seem to change in both character and magnitude. In essence, the apparent formation of 'streamlines' in the water due to the forward motion of the hull introduces forces that were totally absent when the hull was at rest and, as a consequence, the resultant horizontal component of the forces acting on the moving hull is no longer zero, creating a resistance to the hull's forward motion. Logically, at the same time, the resultant vertical component of those same forces affects the displacement of the vessel. Although it is accepted that for the vessel to float on the surface of the water the resultant vertical component of all the forces acting on the hull must still equal the weight of the vessel, the resultant vertical component for a moving vessel is, in fact, considered to be in two parts, the vertical component of the forces due to the boats forward motion and the statical buoyancy of the immersed portion of the hull. Consequently, when a vessel is moving, the 'statical' buoyancy of its hull is generally considered to be less than when the vessel is at rest.

Not surprisingly, because the magnitude of the forces acting on a moving vessel is directly influenced by the configuration of the hull itself, the mathematical determination of the wave-making resistance of an arbitrarily shaped hull will always be an extraordinarily complicated process. The elusiveness of such an outwardly simple goal has challenged elite mathematicians for centuries and, in reality, a wholly theoretical approach to evaluate the wave-making resistance of an arbitrarily shaped hull, using mathematics alone, may never be perfected. The *Component Waveform Theory*, however, adopts a much more fundamental approach, not unlike that of Russell's or Archer's, allowing the hull at any particular design speed to be 'moulded' by the shape of the waves it is forming so as to reduce to a minimum the interaction between the hull and the water particles without the need to calculate the actual magnitude of the forces involved. But, unlike either the *Waveline Theory* or the *Waveform Theory,* the *Component Waveform Theory* is based on the uncommon presumption that as a vessel moves across the surface of the water, at every single point in its path, the vessel's hull creates a circular component wave, depicted in Figure 8-5, similar in many ways to the disturbance made by a raindrop falling onto the water surface.

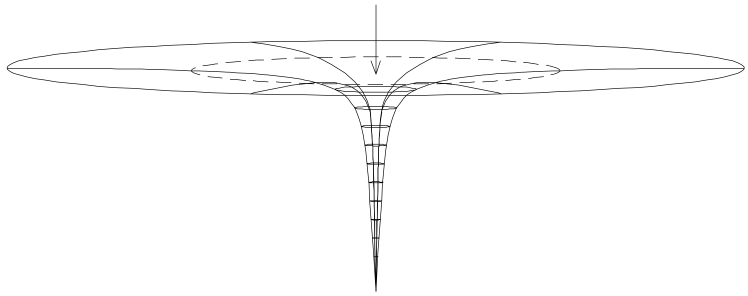


Figure 8-5

However, rather than being caused by a single impact, as in the case of a raindrop, the *Component Waveform Theory* maintains that, in passing a fixed point on the water surface, each cross-section of a hull, from bow to stern, contributes to the displacement of the water particles at that point, combining to generate a component wave in the process. The disturbance is a form of shockwave that is constantly evolving and spreading outward as each underwater cross-section of the hull applies, in turn, a force of unkown magnitude to the water particles to assist in the creation of the wave. For the hull to experience a minimum of resistance in generating the component wave, the forward cross-sections of the hull could, therefore, be assumed to be pushing the water particles aside in some orderly manner as they pass that fixed point on the water surface while the cross-sections aft, as they pass that fixed point, could be assumed to be matching, perfectly, the return of the water particles towards an equilibrium position, a concept similar in that respect to how Russell and Archer envisaged the wave-making process.

Figure 8-6 depicts a vessel approaching a fixed point on the water surface at a constant speed.

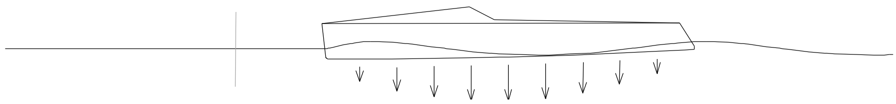


Figure 8-6

Intuitively, the force exerted on the water particles by a cross-section of small dimensions towards the bow of a conventional hull could be expected to be far less than that of, say, a much larger cross-section amidships. As well, from Figure 8-6, it can be readily deduced that, as the vessel moves horizontally across the water surface, each of those cross-sections continues to apply that same force to the water particles at every other point in the vessel's path. Logically, it follows that a vessel moving across the surface of the water at a constant speed must, at any one instant, be impacting the water particles in its path with an infinite number of individual impulses, presumably of different magnitudes, delivered by the infinite number of cross-sections along the hull's entire underwater length. For a vessel moving across the surface of the water, the *Component Waveform Theory* assumes, therefore, that at any one instant every underwater cross-section of the hull is interacting with the water particles and contributing, in some way, to the formation of a separate disturbance, each of those disturbances being an individual component wave. After losing contact with the hull, each component wave, shown complete in Figure 8-5, immediately breaks down and combines with other component waves to form the visible wake.

Unlike the concepts put forward by Russell and Archer, the relationship between a vessel moving across the surface of the water at a particular speed and the separate disturbances caused at any one instant by the infinite number of individual impulses along the vessel's path is not easily visualised. Likewise, the relationship between the distribution of the hull's volume and the vessel's wave-making resistance is, at first glance, equally baffling until it is realised that the forces being applied simultaneously to the water surface at any one instant to produce the resultant wake are identical to the forces depicted in Figure 8-6, applied sequentially in passing a fixed point on the water surface to produce a single component wave at that point. Consequently, it follows that the sum total of the energy being expended by a 'perfectly' shaped hull, that matches exactly the natural movement of the surrounding water particles to generate the visible wake, must therefore be exactly equal to the energy contained within a single component wave. Acknowledgment of the existence of the unseen component waves allows the wave-making resistance of a vessel at any one instant to be viewed, then, not as a measure of the energy being expended by the hull on the surrounding water particles to generate the complexity of waves that is the resultant wake, but as a measure of the energy expended to create one much 'cleaner' component wave before it becomes entangled with others to form that wake.

The maximum energy that a hull can impart to the water particles in its path at any one instant is the total kinetic energy that the hull possesses due to the its forward motion. The energy lost in overcoming surface friction and wind resistance do not enter into that calculation because their effect of constantly impeding the vessel's progress has already been accounted for and it is the resultant speed of the vessel, after all the retarding forces have exercised their influence, which determines the magnitude of the vessel's kinetic energy. Interestingly, observations by the novice in the rowing dinghy seemed to indicate that at slow speeds only a portion of a vessel's kinetic energy is transferred to the water particles, generating small waves on the water surface and leaving sufficient energy to propel the boat forward for a considerable distance after the oars were removed from the water. Nearer 'hull speed', the waves within the wake were bigger and removing the oars had the effect of slowing the boat almost instantly, suggesting that towards 'hull speed' a much higher proportion of a vessel's kinetic energy is constantly being transferred to the water particles. Although the novice was unable to experience planing speeds in the rowing dinghy, if some passing water skiers who had released their tow ropes to slide unassisted to the nearby shore had been observed, the exceptional distance travelled by the skiers without any form of propulsion may have led the novice to the conclusion that at planing speeds again only a portion of a surface vessel's available kinetic energy is transferred to the water particles at any one instant. An experienced sailor might have drawn similar conclusions by contemplating the performance of a modern sailing dinghy in marginal planing conditions, the hull quickly 'dropping off the plane' on a reach when the wind eases slightly but moving steadily to windward with apparent ease at a lower speed in the same fluctuating breeze. Fortunately, for the sailboat designer, there is a simple way of visualising the effect of the instantaneous transfer of kinetic energy from a hull to the water particles at various speeds and, in turn, a method to determine the optimum shape of the hull at each of those speeds to minimise the hull's impact on those particles.

Consider, first, a ball being dropped from different heights onto the water surface, as depicted in Figure 8-7.

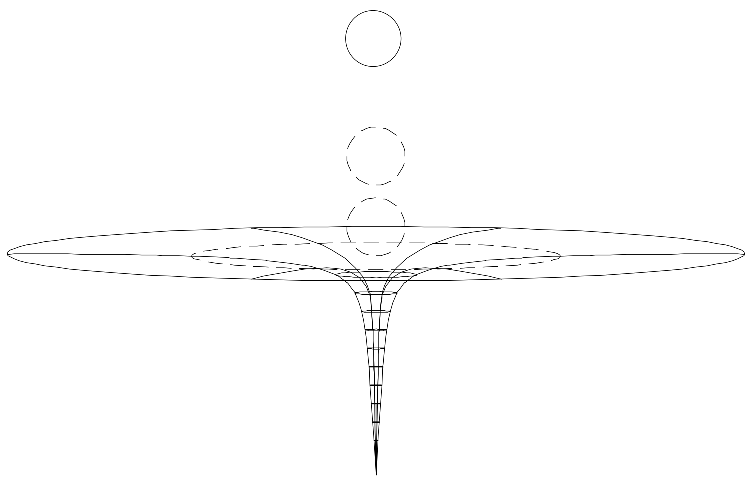


Figure 8-7

Released from a great height, the ball obviously strikes the water surface with tremendous force, causing an exceptionally large splash and clearly displacing a volume of water of weight far greater than its own. At the other extreme, if the ball is let go from a height at which it is already partly submerged and therefore already partially supported by the water, the disturbance of the water surface is barely a ripple. As a matter of interest, it naturally follows that at some point between those two extremes, there is a height from which the ball can be released to displace exactly its own weight of water on impact. In each case, during the collision, the ball imparts to the water particles in its path a specific amount of kinetic energy due to its motion. Apart from the direction in which the ball is travelling, the effect of the ball dropping vertically to impact the water surface with a certain amount of kinetic energy due to its velocity seems not too far removed from the cumulative effect of the cross-sections of a hull moving across the surface of the water and impacting the same water particles at the centre of the disturbance with the same amount of kinetic energy, but from a horizontal direction. Making use of the analogy of the ball being dropped onto the water surface, it now becomes possible to more easily visualise the overall effect of a hull's impact on the surrounding water particles.

Essentially, at the core of the *Component Waveform Theory* is the simple notion that for a conventional hull moving across the surface of the water, the energy being expended by the hull on all the water particles with which it is colliding at any one instant to form the visible wake is exactly equal to the energy transferred to the water particles at any one point on the water surface during the brief moment it takes for the hull to pass. To minimise the interaction between the hull and the water particles at that one point in the vessel's path, the *Component Waveform Theory* contends that the forces exerted by each of the hull's passing cross-sections should be in harmony with the forces at work within the the unseen circular wave that is generated at that point on the water surface, a wave which immediately combines with other such waves to form the visible wake. For the purpose of gaining an insight into the interaction between the hull and the water particles at that one point on the water surface, the energy being expended by the hull in passing that point is also reasoned to be exactly equal to the energy transferred to the water particles by the dropping onto the water surface, from a particular height, an object that could be considered the equivalent of the hull, each situation resulting in the formation of an identical circular wave pattern.

The difficulty of matching the unknown forces applied to the water particles by a hull's passing cross-sections to the equally unknown internal forces at work within the complex circular component wave pattern that is created on the water surface can be lessened somewhat by viewing the formation of a single component wave from a different perspective.

In open water, an object repeatedly dipped in and out of the water at a fixed point on the water surface is regarded as a constant source of disturbance, causing a series of waves which radiate outward in a circular pattern. The amplitude of the circular waves caused by such a disturbance naturally diminishes as the waves travel further from the source, the waves are continually expanding and covering a larger area as they move outward from the centre of the disturbance. But, the total energy being expended by the internal forces at work within each of the waves at any given distance from the centre of the disturbance remains constant as the waves advance. For a disturbance similar to that described above but confined to within a parallel-sided channel, rather than occurring in open water, a force equal to that applied to the open water surface would cause, instead, a series of transverse waves moving away from the disturbance in opposite directions, as depicted in Figure 8-8. At any given distance from the centre of each of the disturbances, the internal forces at work within the transverse waves confined to the parallel-sided channel would be expending exactly the same amount of energy as the corresponding forces within the circular waves in open water. However, unlike circular waves on an open water surface, transverse waves in a parallel-sided channel retain their amplitude and shape as they progress, allowing a less complicated analysis of the waves and how the object creating the waves might be interacting with the water surface. Information such as the volume of water displaced to generate the waves, and the wavelength of the waves that are formed, can be more easily extracted.

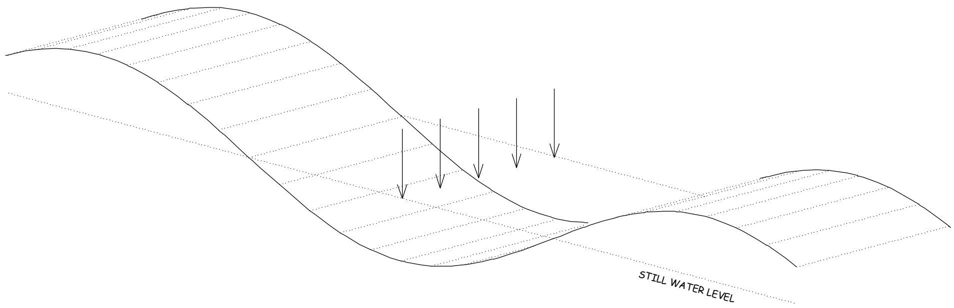


Figure 8-8

To address the fundamental question of how knowledge of the component waves helps to shape a hull, the *Component Waveform Theory* assumes that the passing of a hull's cross-sections at a fixed point on the water surface delivers a sequence of impulses and each of those impulses contributes to the formation of a circular component wave at that point, in a manner not unlike an object being dipped in and out of the water, the essential difference being that the hull impacts the water particles at the fixed point only the once in passing. For a hull to be in perfect harmony with the circular component waves being produced, the *Component Waveform Theory* also presumes that because the total force applied by each cross-section of the passing hull is reasoned to be directly proportional to the immersed area of the cross-section, in order to match the energy being expended by the internal forces at work within each component wave the immersed area of each of the hull's cross-sections should match exactly the corresponding sectional area of the component wave as it is being formed. Alternatively, it could simply be argued that if each cross-section of the hull was deemed to have a finite thickness, in passing a fixed point on the water surface each cross-section would naturally displace its own volume of water, directly contributing to the formation of the circular component wave. By analysing the circular component waves from the perspective gained by confining one component wave to a parallel-sided channel, the transverse waves in the channel could be expected to reveal information as to how each circular component wave is produced and provide a snapshot of the sectional areas of the component wave as the wave is being formed, the key to determining the properties of the unique curve that represents the optimal cross-sectional areas of the hull. The challenge is how to determine the shape of the component waves produced by a vessel of known waterline length and displacement, travelling at a given velocity and, as a consequence, how then to determine the appropriate curve of areas of the hull to minimise the hull's wave-making resistance.

The following chapters explain procedures that enable the calculation of the longitudinal displacement of a hull at any one velocity by application of the *Component Waveform Theory.* Adopting a perfect, non-viscous, incompressible, elastic fluid of infinite depth and extent, as Isaac Newton had assumed for his theory of resistance and František Gerstner for the derivation of his *Trochoidal Wave Theory*, the profile of the transverse wave pattern within the parallel-sided channel is accepted as being trochoidal. To demonstrate the basic principles of the theory, however, the trochoid will be substituted for the time being by the much simpler sine curve, which is a very close approximation to the trochoid when the amplitude of the curve is very small in comparison to the wavelength. As well, although the the *Component Waveform Theory* is equally applicable to all hulls at most speeds, from 'cruising' speed, a speed-length ratio of about R=1.7, to high planing speeds, for explanatory purposes the circumstances of a hull moving at 'hull speed', below 'hull speed' and above 'hull speed' are considered separately.

HULL SPEED

The *Component Waveform Theory* is founded on the presumption that minimising the resistance caused by the creation of waves necessitates that a vessel's hull present to the fluid a smooth progression in the longitudinal distribution of its displacement. Although the optimal distribution of that displacement is considered to be determined directly by the shape of the circular component waves which are generated at every point in the vessel's path and which then combine to form the visible wake, the precise configuration of each component wave is, in turn, clearly dependent on the static displacement of the vessel, the vessel's waterline length and the speed at which the vessel is travelling. Consequently, at each point in its path every surface vessel generates a circular component wave of unique proportions for each speed at which it travels and, being a rigid structure with a fixed waterline length and displacement, a hull cannot be designed to conform to the properties of those waves for any more than one particular speed. At all other speeds the hull's proportions will be at variance with those of the component waves being generated, resulting in an increase in wave-making resistance and a hull design of less than maximum efficiency.

Adding to that complexity, for a vessel moving across the surface of the water at sub-planing speeds there is a definite distinction between a vessel's static displacement and the volume of water that is actually displaced by the vessel's forward motion. A mistaken belief, sometimes loosely expressed in boating publications, is that the waves visible within a vessel's wake at displacement speeds are formed by the vessel continually pushing aside its own weight of water. Although that perception may seem quite plausible, such a misunderstanding could not be further from the truth. While a vessel at rest does, in fact, displace its own weight of water, earlier observations of the performance of a rowing dinghy led to the conclusion that the volume of water pushed aside by the forward motion of a vessel at sub-planing speeds is somehow proportional to the kinetic energy imparted to the water particles by the hull. At slow speeds the waves within a vessel's wake are small and cause minimal resistance, but beyond 'cruising' speed, a speed-length ratio of about R=1.7, the waves begin to increase markedly in size and the performance of a vessel becomes more seriously affected. Observations of a ball dropped onto the water surface from various heights seemed to infer that as a vessel proceeds at slow speeds across the water surface the amount of energy transferred to the water particles at any one instant is small and the size of the component waves that ultimately form the wake are also correspondingly small. Further observations suggested that as the velocity of a vessel is increased, so too is the amount of energy transferred to the water particles, creating component waves of greater wavelength and volume. One exceptional conclusion derived from the ball experiment, however, was that, for a vessel moving across the surface of the water, there is one specific speed at which the amount of energy being continually transferred to the water particles is such that the volume of water pushed aside in the formation of a single component wave at each point in its path does, in fact, equal the static displacement of the vessel. The uniqueness of that particular circumstance warrants closer examination and suggests itself as a starting point for the mathematical investigation into the effects of wave-making generally.

Figure 9-1 represents an exaggerated profile of a component wave caused by a full-width disturbance in a parallel-sided channel. The 'disturbance' in this particular case is created by dropping the equivalent of a modern light-displacement sailing dinghy such as an NS14 onto the water surface from such a height that the impact triggers the displacement of a volume of water equal in weight to the static displacement of the fully-laden boat. To simplify the analysis, rather than dropping the actual boat onto the surface in open water, a block of wood of neutral density, equivalent in displacement and LWL to the fully laden boat, is dropped into a parallel-sided channel of the same width as the block which, in turn, is shaped to have a profile that is in harmony with the natural flow of water away from the centre of the disturbance. By definition, the profile of the block of wood is the hull's 'curve of areas'.

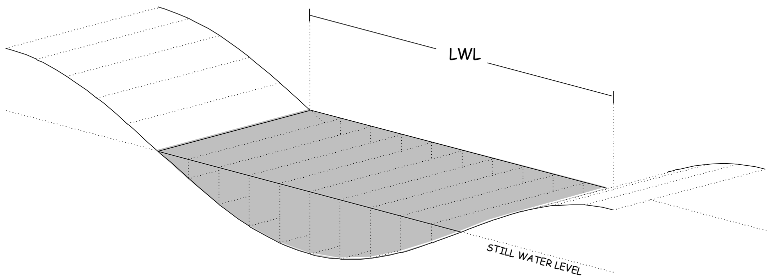


Figure 9-1

S_1 and S_2 in Figure 9-2 are points located on the surface of the wave at each end of the hull's load waterline and the horizontal distance between S_1 and S_2 is the hull's load waterline length, LWL. The shaded area below the line S_1S_2 represents the static displacement of the boat. By inspection, assuming the profile of the wave that is formed to be a sine curve, rather than a trochoid, the wavelength of the wave is equal to twice the length of the boat's waterline and, according to the *Component Waveform Theory*, the cross-sectional areas of the hull should match exactly the cross-sectional areas of the wave below still water level. The question, of course, is at what speed does the boat create such a component wave, not when 'dropped' but when moving across the water surface?

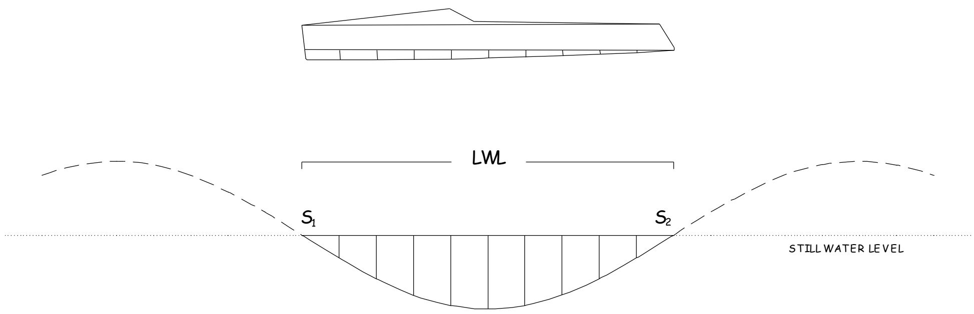


Figure 9-2

During the momentary contact between a hull moving across the surface of the water and the water particles in its path at any one point, for the circumstance depicted in Figure 9-1 in which the volume of water physically pushed aside is equal to the static displacement of the fully-laden boat, the *Component Waveform Theory* logically assumes, after the ball experiment, that the entire kinetic energy of the boat is transferred to those water particles, setting them in motion. The theory also assumes that the water particles, having been set in motion, initially form a single wave which, under the given parameters, immediately breaks down after losing contact with the hull, forming, in turn, a group of smaller waves, the actual shapes of which are inconsequential for the purpose at hand. Significantly, though, as the impulse of the collision is relayed outward from the centre of the disturbance, the transferred energy is considered to advance at the group velocities of the waves so formed, both during and after contact with the hull, not at the velocities of the individual waves.

For his solitary wave, John Scott Russell argued that an impulse such as depicted in Figure 9-1 causes more than just a deformation of the water surface and that what he had consistently observed in his experiments involved an actual mass transport of water, as illustrated earlier in Figure 4-1. There is little reason to believe that should not be the case since the hull is physically pushing water particles aside and so, based on Russell's observations, the *Component Waveform Theory* assumes also that the impact of the hull on the water particles does, in fact, cause an initial mass transport of water away from the disturbance.

For the particular circumstance, therefore, in which a vessel is not dropped onto the water surface, as depicted in Figure 9-1, but travels horizontally past a fixed point on the water surface at such a speed that the cumulative impact of the hull's passing cross-sections on the water particles causes the volume of water displaced at that point to be equal in weight to the vessel's static displacement, the *Component Waveform Theory* assumes that the total kinetic energy of the vessel is not only transferred to the water particles to form a component wave but, in so doing, sets in motion a volume of water equal in mass to that of the vessel.

František Gerstner's *Trochoidal Wave Theory* showed that waves on the surface of the water have equal amounts of kinetic and potential energy. Therefore, it follows that the mass of water physically displaced in the form of a wave during contact with the hull's passing cross-sections at a fixed point in the vessel's path gains equal amounts of both kinetic and potential energy, so that during contact:

$$KE_{vessel} = (KE + PE)_{displaced\ \ water}$$

leading to:

$$KE_{vessel} = (2KE)_{displaced\ \ water}$$

$$(\tfrac{1}{2}mv^2)_{vessel} = 2\,(\tfrac{1}{2}mv^2)_{displaced\ \ water}$$

For the circumstance depicted in Figure 9-1, however, the mass of water displaced to create a component wave at each point in the vessel's path is known to be equal to the static displacement of the vessel, so that:

$$m_{vessel} = m_{displaced\ \ water}$$

resulting in:

$$v^2{}_{vessel} = 2\,v^2{}_{displaced\ \ water}$$

During contact between the hull and the water particles at a particular point, the *Component Waveform Theory* maintains that the impulse of that impact is relayed outward from the centre of the disturbance, initially in the form of a single wave. Therefore, the energy of the water particles and, logically, the volume of water physically displaced at that point in the vessel's path, is reckoned to advance not at the velocity of that component wave but at the 'group' velocity of the wave system, which is accepted as being half the velocity of the component wave, so that:

$$v^2{}_{vessel} = 2\left(\frac{v_{component\ \ wave}}{2}\right)^2$$

$$v^2{}_{vessel} = 2\left(\frac{\sqrt{\frac{g\lambda}{2\pi}}}{2}\right)^2$$

$$v_{vessel} = \sqrt{\frac{g\frac{\lambda}{2}}{2\pi}}$$

Remembering that the sine curve adopted for the profile of the component wave is an approximation only, the calculations show that the velocity of the boat in this particular instance is equal to the velocity of a wave having half the wavelength of the component wave or, more to the point, the velocity of the boat is equal to the velocity of a wave having a wavelength approximately equal to the boat's waterline length. In other words, the boat depicted in Figure 9-1 is travelling at 'hull speed' and the profile of the resultant transverse waves within the wake is as illustrated in Figure 9-3, the transverse waves within the visible wake having a wavelength of approximately half that of the 'invisible' component waves.

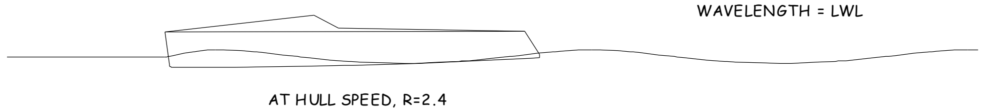


Figure 9-3

Immediately, the dissimilarity of the approach taken by the *Component Waveform Theory* to that of Colin Archer's *Waveform Theory* becomes evident.

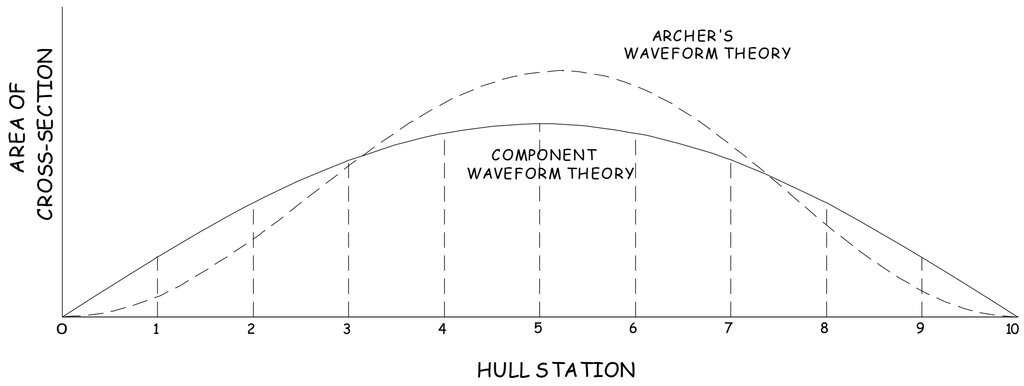


Figure 9-4

Understandably, Archer, like Russell before him, had made use of the visible wave pattern of the resultant wake at 'hull speed', as illustrated in Figure 9-3, to shape his ideas on how to minimise wave-making resistance, basically distributing the underwater volume of the hull over the full length of a wave having a wavelength equal to the hull's waterline length, as shown in Figure 6-2. In contrast, for a vessel at 'hull speed' the *Component Waveform Theory* distributes the underwater volume of the hull over a component wave having a wavelength equal to approximately twice the waterline length of the hull but only over that portion of the wave below still water level. Figure 9-4 constrasts the differences between the curve of areas derived from the *Component Waveform Theory* for a hull at 'hull speed' and that by Archer's *Waveform Theory*, the former producing a hull with 'fuller' ends and a smaller midship section. The alternate use of a more accurate trochoidal profile for the component wave to determine the hull's cross-sectional areas accentuates those differences.

Although not absolutely relevant to the task in hand, a brief analysis of the visible wake at 'hull speed' is of interest.

William Froude, when he first began towing model boats on his local waterway, soon recognised the fact that the wake made by a vessel always appears static when viewed from above or, in other words, the entire wave pattern that is the resultant wake keeps pace with the vessel creating the wake as it moves across the surface of the water. Perhaps the most notable consequence of the wave-making phenomenon is that the waves within a vessel's wake have, by necessity, wavelengths that enable them to keep pace with the vessel. With knowledge of the existence of the component waves, however, the fact that the resultant wake always keeps pace with the vessel so that the wave pattern appears static when viewed from above is not surprising. Briefly, when any vessel moves across the surface of the water at a constant speed, the repetitive impact of the hull on the water particles creates identical component waves, one after the other at exactly the same rate, at every point in the vessel's path. When those component waves ultimately combine to form the visible wake there is little reason to suspect that the resultant wave pattern should ever fluctuate. Relative to the hull, the wake would instinctively be expected to be static and, as Froude observed, that appears to be the case.

William Thomson later made use of Froude's discovery that the wake appears static, relative to the hull, to determine mathematically the angle of the V-shaped pattern made by the diverging bow waves within the wake. Thomson considered a point source of disturbance, rather than an actual hull, to calculate a wake angle of 19°28' to the direction of travel, an angle which he maintained also remained constant, regardless of the velocity of the disturbance. To an observer, however, the actual angle of the outer boundaries of the wake relative to a vessel's direction of travel is not easily defined but, from casual observation of a vessel moving from displacement to planing speeds, the angle appears to be variable, not constant, in conflict with Thomson's calculations and inviting further investigation.

For the case of a vessel travelling at 'hull speed' it was previously assumed by the *Component Waveform Theory* that during the brief period of contact between the hull and the water particles with which it collided at a particular point, the hull transferred to the water particles all of its kinetic energy, creating, in theory, a single component wave, as depicted in Figure 8-5. Logically, being a single wave, the total energy contained within the component wave is equal to the kinetic energy imparted to the water by the hull. To maintain a progression of similar waves travelling outward from the disturbance would require a continuing input from the initial energy source, such as could be achieved by repeatedly 'dipping' the equivalent of the hull in and out of the water at the same point. When the hull impacts the water particles only once, though, the initial wave inevitably breaks down after losing contact with the hull. Only under exceptional circumstances, such as in the confines of the canal where Russell conducted his first experiments, can a component wave manage to sustain itself as a solitary wave. In deep open water the component wave collapses into a complex group of waves which continues outward from the initial disturbance. Within that complex group researchers have, in fact, sometimes detected smaller solitary waves travelling outward far ahead of a moving vessel but, overall, the resultant wave group produced by a single impulse on the water surface consists predominantly of a series of ordinary oscillating waves which, in the case of a vessel moving horizontally across the surface, ultimately combine with similar waves being formed at every other point in the vessel's path, resulting in the familiar visible wake.

After a vessel travelling across the surface of the water has moved on, at every point in its path it is only the kinetic energy of the initial wave system created by the hull's passing that can advance the wave motion into undisturbed water. As the initial wave system breaks down and advances into calm water, the water particles at the leading edge are obliged to provide sufficient energy to not only put undisturbed particles in motion but to also lift those particles above their 'at rest' positions. More precisely, the kinetic energy that the particles at the leading edge of the initial wave system have because of their motion is required to provide both the kinetic energy and the potential energy of the adjacent particles in the undisturbed water ahead. Kinetic energy is transferred as the particles collide but clearly, the kinetic energy of a lone particle within the initial wave system is insufficient to provide an adjacent water particle with an equal amount of kinetic energy plus the energy needed to raise that particle above its 'at rest' position in calm water. By deduction, the newly formed wave group, comprised predominantly of ordinary oscillating waves containing equal amounts of potential and kinetic energy, can only advance at approximately half the velocity of its energy source, the initial wave system.

Previously it was deduced that during impact:

$$v^2{}_{vessel} = 2\, v^2{}_{displaced \ \ water}$$

leading to:

$$v_{displaced \ \ water} = \frac{v_{vessel}}{\sqrt{2}}$$

Therefore, after the displaced water has lost contact with the hull and collapsed into a wavetrain of mostly ordinary oscillating waves advancing outward from the point of the initial impulse at half the velocity of the initial wave system, by simple deduction, the velocity of the advancing wavetrain can be taken to be:

$$v_{wavetrain} = \frac{v_{vessel}}{2\sqrt{2}}$$

In other words, at 'hull speed' a vessel travels at approximately $2\sqrt{2}$ times the velocity of the predominant wavetrain that expands outward from the centre of each disturbance created by the vessel at every point in its path.

The angle between the direction of travel of the source and either of the two straight lines which approximately define the outer limits of the wake, as illustrated in Figure 9-5, if represented by the value Ψ, may well be given by:

$$\tan\Psi = \frac{v_{wavetrain}}{v_{vessel}} = \frac{1}{2\sqrt{2}}$$

$$\Psi = 19°28'$$

By application of the *Component Waveform Theory*, the V-shaped boundary of a vessel's wake at 'hull speed' could, therefore, be said to have an angle of 38°56', as previously calculated by Thomson for a point source of disturbance.

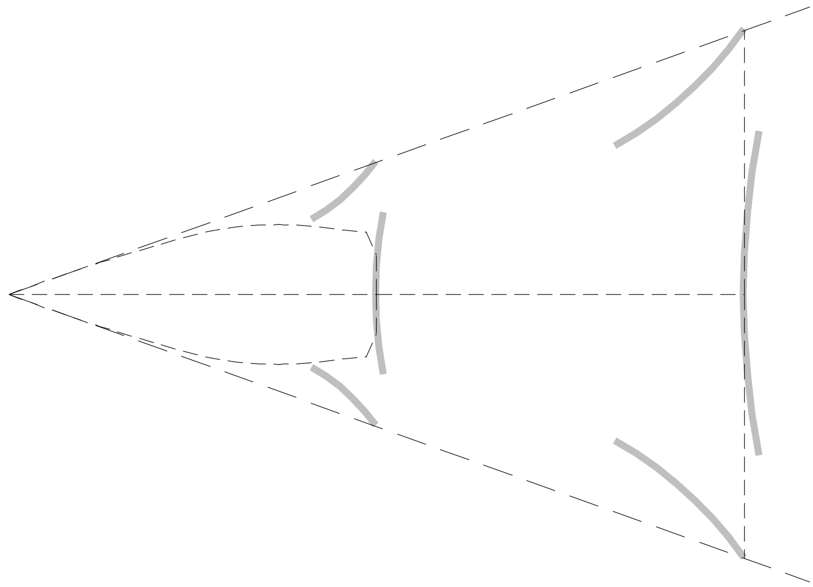


Figure 9-5

Using similar logic, if the waves at the outer limits of the wake are reckoned to have an outward velocity twice that of the wavetrain of which they are part and a forward velocity equal to that of the vessel, those diverging crests formed off the bow would need to travel at a direction of 35°16' to the direction of travel of the source. In reality, though, the circular patterns of waves, which emanate from the infinite number of disturbances caused by individual impulses along a vessel's path and immediately merge to form the visible wake, are, in themselves, extremely complex, being comprised of waves of various lengths and speeds. Thomson, having conducted experiments on the disturbance produced by a single impulse on the surface of the water, was well aware of that fact and would have been equally aware that any simple approach to determine the angle of the outer limit of a vessel's wake can never provide an outcome that is anything more than an estimate of what actually occurs.

BELOW HULL SPEED

For a vessel moving across the surface of the water at any speed below 'hull speed', the *Component Waveform Theory* assumes that only a portion of the total kinetic energy of the vessel is transferred to the water particles with which it collides at every point in its path, forming a component wave and setting in motion a volume of water of mass less than the static displacement of the vessel at each of those points.

Figure 10-1 represents an exaggerated profile of a component wave caused by a full-width disturbance in a parallel-sided channel. The 'disturbance' in this particular case is created by dropping the equivalent of the NS14 sailing dinghy onto the water surface from such a height that the impact triggers the displacement of a volume of water of weight less than the static displacement of the fully-laden boat. To simplify the analysis, a block of wood of neutral density, equivalent in displacement and LWL to the fully laden boat, is dropped into a parallel-sided channel of the same width as the block which, in turn, is shaped to have a profile that is in harmony with the natural flow of water away from the centre of the disturbance. By definition, the profile of the block of wood is the hull's 'curve of areas' for the given parameters.

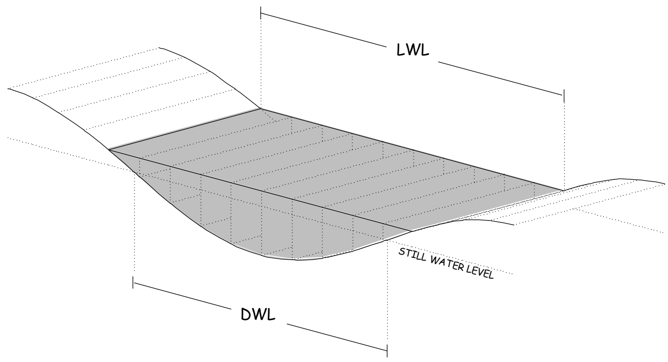


Figure 10-1

As depicted in Figure 10-1, on impacting the water surface with less kinetic energy than at 'hull speed', the *Component Waveform Theory* assumes that not only is less water displaced by the impact but, unlike at 'hull speed', a portion of the hull is momentarily supported by the wave which has been formed, supposedly causing the 'dropped' boat to come to a stop above its normal 'at rest' position.

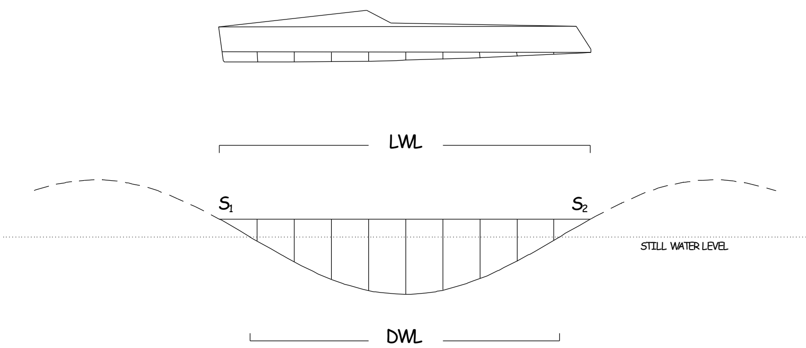


Figure 10-2

S_1 and S_2 in Figure 10-2 are points located on the surface of the component wave at each end of the hull's load waterline and the horizontal distance between S_1 and S_2 is the hull's load waterline length, LWL. The shaded area below the line S_1S_2 represents the static displacement of the boat whereas the portion of the shaded area below still water level represents the volume of water displaced in the form of a wave due to the boat's motion. The effective length over which the hull is actually displacing water to create the component wave is the length of the hull at still water level, labelled DWL to represent the 'design' waterline length of the hull as opposed to the actual waterline length. Obviously, for all speeds below 'hull speed' the DWL is shorter than the LWL.

By inspection, assuming the profile of the wave to be a sine curve, rather than a trochoid, the wavelength of the wave in Figure 10-2 is equal to twice the length of the boat's design waterline and, according to the *Component Waveform Theory*, the cross-sectional areas of the hull should match exactly the cross-sectional areas of the wave below the line S_1S_2. The question again, of course, is at what speed does the boat create such a component wave, not when 'dropped' but when moving across the water surface?

During the momentary contact between a hull moving horizontally across the surface of the water and the water particles in its path at any one point, for the circumstance depicted in Figure 10-1 in which the volume of water physically pushed aside is less than the static displacement of the fully-laden boat, the *Component Waveform Theory* logically assumes, after the ball experiment, that only a portion of the kinetic energy of the boat is transferred to those water particles, setting them in motion. More particularly, the *Component Waveform Theory* assumes that during the impact of the hull on the water particles at each point in the vessel's path at speeds below 'hull speed', only the kinetic energy of the equivalent of that part of the vessel which is below still water level in Figure 10-1 is transferred to the water particles to create the component wave. Those water particles are immediately set in motion and, as happened at 'hull speed', the impulse of the collision is relayed outward from the centre of the disturbance, so that during contact:

$$KE_{hull\ below\ SWL} = (KE + PE)_{displaced\ \ water}$$

Again, the mass of water physically displaced during contact with the hull's passing cross-sections is in the form of a wave and gains equal amounts of both kinetic and potential energy, so that:

$$KE_{hull\ below\ SWL} = (2KE)_{displaced\ \ water}$$

$$(\tfrac{1}{2}mv^2)_{hull\ below\ SWL} = 2\,(\tfrac{1}{2}mv^2)_{displaced\ \ water}$$

But, for speeds below 'hull speed' the *Component Waveform Theory* has reasoned that the mass of the water displaced to create a component wave at each point in the vessel's path is equal only to the displacement of that part of the hull which is below still water level, as depicted in Figure 10-2, so that:

$$m_{hull\ below\ SWL} = m_{displaced\ \ water}$$

leading to:

$$v^2{}_{vessel} = 2\,v^2{}_{displaced\ \ water}$$

resulting in:

$$v^2{}_{vessel} = 2\left(\frac{v_{wave}}{2}\right)^2$$

$$v^2{}_{vessel} = 2\left(\frac{\sqrt{\frac{g\lambda}{2\pi}}}{2}\right)^2$$

$$v_{vessel} = \sqrt{\frac{g\frac{\lambda}{2}}{2\pi}}$$

Remembering that the sine curve adopted for the profile of the component wave is an approximation only, the calculations show that the velocity of the boat in this particular instance is equal to the velocity of a wave having half the wavelength of the component wave or, more to the point, the velocity of the boat is equal to the velocity of a wave having a wavelength approximately equal to the boat's design waterline length.

Logically, if the speed of the boat is progressively reduced, a 'minimum' design speed consistent with the *Component Waveform Theory* could be anticipated when the wavelength of the component wave becomes equal to the load waterline length of the hull, as in Figure 10-3. At speeds lower than that 'minimum', the waterline length of the hull would always exceed the wavelength of the component wave being formed and, as a consequence, the cross-sections of the hull would never be able to conform exactly to the shape of the wave, thereby having a detrimental effect on the wave-making resistance of the hull at those low speeds.

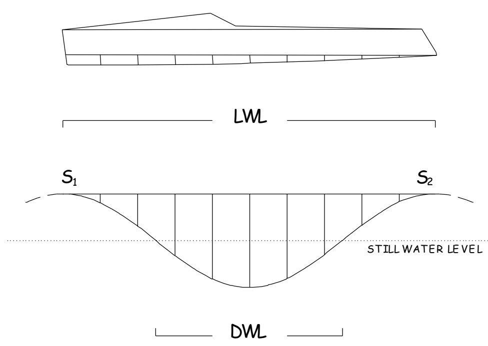


Figure 10-3

Figure 10-3 depicts the boat travelling at the 'minimum' design speed consistent with the application of the *Component Waveform Theory.* As shown, at that particular speed the wavelength of the component wave has shortened to equal the load waterline length of the hull and, logically, the design waterline has also reached its 'minimum' length. By calculation, using the formula derived previously, the velocity of the boat at the 'minimum' design speed consistent with the application of the *Component Waveform Theory* is, therefore, equal to the velocity of a wave having a wavelength approximately equal to half the length of the hull's load waterline. In other words, at the 'minimum' design speed the boat is operating at a speed-length ratio of approximately R=1.7 and the profile of the resultant transverse waves within the wake is as illustrated in Figure 10-4.

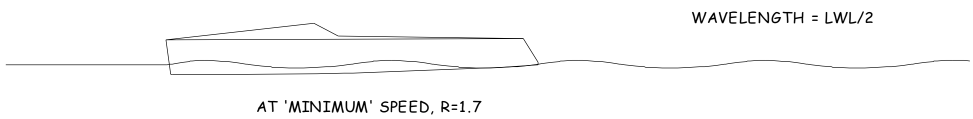


Figure 10-4

Comparing, once again, the curve of areas produced by the *Component Waveform Theory* for a speed-length ratio of R=1.7 with that of Colin Archer's *Waveform Theory*, as illustrated in Figure 10-5, the similarities are obvious. In fact, for all intents and purposes the curves are practically identical. From the perspective of the *Component Waveform Theory* it is of little wonder, then, that the application of Archer's *Waveform Theory* produced wholesome sailing yacht designs that performed exceptionally well within the 'normal' speed range of ballasted sailboats of that period in moderate winds. Equally apparent, though, is that while those same designs may have been capable of reaching, or even exceeding, 'hull speed' under exceptional circumstances, consistently high speeds could not possibly be expected without excessive wave-making resistance. Although appropriate in its day and generally correct in its approach to the problem of minimising wave-making resistance, Archer's *Waveform Theory* was based, unfortunately, on a flawed hypothesis that inevitably led to an equally flawed conclusion. Rather than improving the design of sailing yachts for speeds approaching 'hull speed', as intended, unwittingly Archer's *Waveform Theory* was, in fact, more suited to the 'comfortable' cruising speeds of the sailing yachts of that era, typically in the the vicinity of the speed-length ratio R=1.7. However, although discounted by the early years of the 20th century as a method to minimise wave-making resistance at speeds approaching 'hull speed', Archer's *Waveform Theory* has since continued to provide sailboat designers with a starting point from which to evolve more efficient designs, a fact which is not surprising in light of the *Component Waveform Theory*.

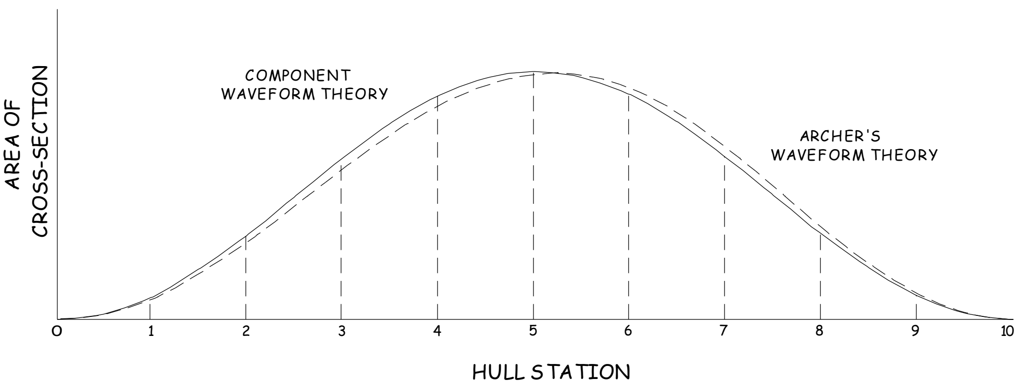


Figure 10-5

In practice, contrary to the relatively 'fixed' approach of Archer's *Waveform Theory,* application of the more flexible *Component Waveform Theory* allows the generation of a specific curve of areas for any chosen displacement speed between the 'minimum' speed-length ratio of R=1.7 and 'hull speed', R=2.4. The area curves derived by the *Component Waveform Theory* over that speed range reveal a gradual transition from the 'fine-ended' shape depicted in Figure 10-5 for low displacement speeds to the 'full-bodied' curve depicted in Figure 9-4, more suited to speeds approaching the 'hull speed' barrier. Although the areas bounded by the respective curves and the horizontal axes are identical and represent the static displacement of the boat, the volume of water actually displaced in the form of a wave by the movement of the hull across the surface of the water at each of those speeds, according to the *Component Waveform Theory,* is markedly different, a claim that is seemingly borne out by performance on the water.

At a speed-length ratio of R=1.7, as depicted in Figure 10-3, the *Component Waveform Theory* maintains that in moving across the surface of the water the boat is displacing only a portion of its static displacement in forming each component wave. Not only that, the *Component Waveform Theory* asserts that, in displacing a lesser amount of water to form the component waves at that particular speed, the hull gains support from the component waves that are continually being formed and is thereby held marginally above its 'at rest' position, effectively allowing the boat to retain a portion of its kinetic energy as potential energy. On the water, a boat such as the NS14 dinghy, moving at a speed-length ratio of about R=1.7, actually does feel 'lighter' and more responsive than at other displacement speeds. Minor fluctuations in wind strength seem not to overly affect the boat's immediate performance, due, no doubt, to the energy held in reserve. In contrast, the *Component Waveform Theory* maintains that in moving across the surface of the water at 'hull speed' a vessel continually displaces the whole of its static displacement in the formation of the component waves, from which the hull gains no support, effectively preventing the vessel from retaining any of its kinetic energy as potential energy. On the water, many sailboats do feel 'heavier' and less responsive at speeds approaching 'hull speed', often giving the impression of being sucked into a hole which they are themselves creating, very much in keeping with the *Component Waveform Theory's* explanation of how the wake is formed. In marginal conditions, if 'hull speed' can be achieved it is not easily maintained, no doubt due to the lack of energy held in reserve at that speed.

Figure 10-6 constrasts the extremes in the possible shapes of the area curves for displacement speeds derived using the *Component Waveform Theory*, showing the specific curves for the speed-length ratios of R=1.7 and 'hull speed', R=2.4. For the range of speeds between those two 'extremes', there is a gradual transition in the shape of the area curves from the 'fine-ended' curve depicted in Figure 10-5 for a speed-length ratio of R=1.7 to the 'full-bodied' curve more suited to 'hull speed', R=2.4.

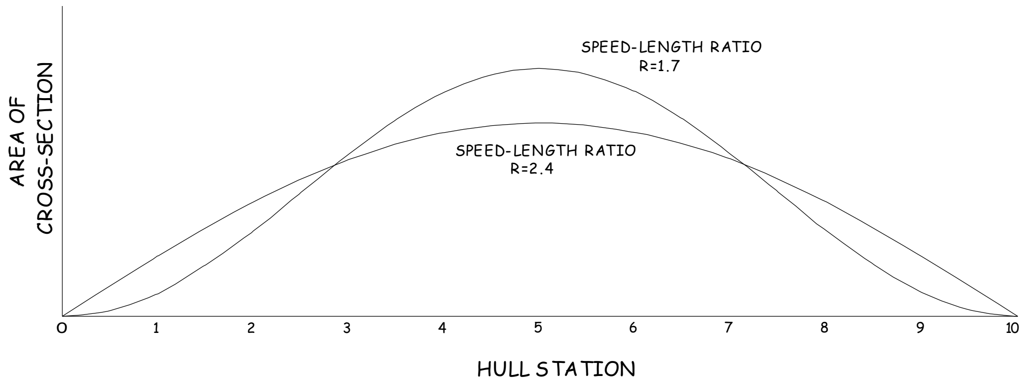


Figure 10-6

Finally, as an aside, the visible wake of a vessel travelling at speeds below 'hull speed' is, according to the *Component Waveform Theory*, subject to exactly the same considerations as at 'hull speed', resulting in a constant wake angle of 19°28' to the direction of travel, at least above a speed-length ratio of R=1.7, in agreement with William Thomson's calculations.

SEMI-PLANING AND PLANING

In the preceding chapters, the *Component Waveform Theory* maintained that, for a vessel moving across the surface of the water between a speed-length ratio of R=1.7 and 'hull speed', only a portion of the total kinetic energy of the vessel is transferred to the water particles with which it collides at every point in its path, thereby setting in motion at each of those points a volume of water of mass less than the static displacement of the vessel. At all times the vessel remained fully supported by the water but the volume of water pushed aside, in the form of a component wave, was claimed by the *Component Waveform Theory* to be directly proportional to the relative speed of the vessel until ultimately, on reaching 'hull speed', the total kinetic energy of the vessel is transferred to the water particles with which it collides at every point in its path and the volume of water displaced to form each component wave is equal to the static displacement of the vessel. Accordingly, above a speed-length ratio of approximately R=1.7, the theory assumed that there is a smooth progression of the circumstances surrounding the formation of the wake of a surface vessel as the speed of the vessel is increased towards 'hull speed'. Immediately beyond 'hull speed', however, the *Component Waveform Theory* contends that there is a well-defined break in those circumstances which is reflected in both the performance of the vessel and the pattern of the resultant wake. Nevertheless, in contrast with conventional design methods, the approach taken by the *Component Waveform Theory* to derive a curve of areas for a vessel moving across the water surface at any speed above a speed-length ratio of R=1.7, including planing speeds, remains consistent throughout the entire speed range.

From the ball experiment it became apparent that, if dropped from a great height, a ball impacts the water surface at high speed and its associated kinetic energy causes such a splash that the ball momentarily displaces more than its own weight of water. Adopting the same approach for a hull, Figure 11-1 represents an exaggerated profile of a component wave caused by a full-width disturbance in a parallel-sided channel. The 'disturbance' in this particular case is caused by dropping the equivalent of the NS14 sailing dinghy onto the water surface from such a height that the impact triggers the displacement of a volume of water of weight greater than the static displacement of the fully-laden boat.

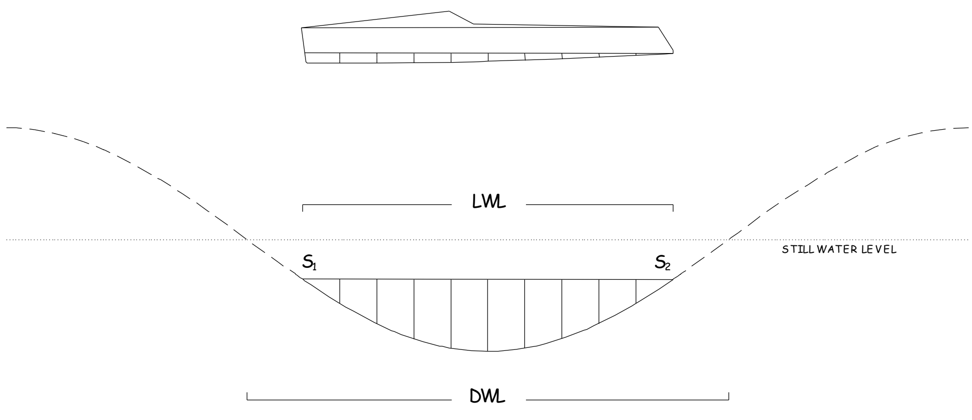


Figure 11-1

As depicted in Figure 11-1, on impacting the water surface with more kinetic energy than at 'hull speed', not only does the 'dropped' boat displace more than its own weight of water but, unlike at 'hull speed', the boat does not come to rest until below the level at which it floats when stationary. S_1 and S_2 in Figure 11-1 are points located on the surface of the component wave at each end of the hull's load waterline and the horizontal distance between S_1 and S_2 is the hull's load waterline length, LWL. The shaded area below the line S_1S_2 represents the static displacement of the boat whereas the total area bounded by the curve and still water level represents the greater volume of water actually displaced in the form of a wave due to the boat's downward motion. The effective length over which the hull is displacing water to create the component wave is the length of the wave intercept at still water level, labelled DWL to represent the design waterline length of the hull as opposed to the actual waterline length. Under the given circumstances, the DWL is necessarily longer than the LWL.

For a vessel travelling across the surface of the water, Figure 11-1 gives every indication that at speeds greater than 'hull speed' the total kinetic energy of the vessel is transferred to the water particles with which it collides at every point in its path, setting in motion, at each of those points, a volume of water of weight greater than the static displacement of the vessel. If the speed of the vessel was progressively increased, such logic suggests that the volume of water displaced would also increase, causing the hull to 'sink' lower and lower into a trough of its own making. In practice, a vessel with a low-speed displacement hull attempting to exceed 'hull speed' strives to do exactly what is implied by Figure 11-1, transfer all of its kinetic energy to the water particles by displacing more than its own weight of water, but is incapable of doing so for one simple reason, the vessel is fully supported by the water and, as was discovered by Archimedes, unless acted upon by an external vertical force, a floating object cannot displace more, or less, than its own weight of fluid. Consequently, a low-speed displacement hull attempting to exceed 'hull speed' cannot possibly achieve what is implied by Figure 11-1 but, instead, is practically restricted to 'hull speed' as an upper speed limit, unable to convert any surplus driving force into component waves of longer wavelength. Designing a vessel to travel across the surface of the water at speeds greater than 'hull speed' requires, therefore, a review of the factors that influence the wave-making process.

Referring, once again, to the ball experiment, if the ball is to be dropped from a great height to impact the water surface so that it momentarily displaces only its own weight of water, and no more, clearly the ball must be restrained in some way so that its fall can be brought to a halt at some point above the ball's stationary floating position. By restraining the ball, on impact with the water surface a lesser portion of the ball is immersed than when floating 'at rest' yet those immersed sections have sufficient kinetic energy due to their high velocity to displace the ball's full weight of water. Making use of that analogy and the fact that a surface vessel must always remain fully supported by the fluid, it becomes apparent that for a vessel moving across the surface of the water at any speed above 'hull speed', only a portion of the vessel's hull need be immersed to displace, at each point in the vessel's path, a volume of water equal to the vessel's static displacement. Consequently, for a vessel moving across the surface of the water at any speed above 'hull speed', the *Component Waveform Theory* asserts that the vessel's hull must be shaped to enable the vessel to rise above its stationary level so that the hull is only partly immersed and, as a consequence, only a portion of the kinetic energy of the vessel is transferred to the water particles with which it collides at every point in its path, setting in motion, in the form of a component wave at each of those points, a volume of water of weight equal to the static displacement of the vessel.

In assuming that the volume of water displaced by the forward motion of an appropriately shaped hull at speeds above 'hull speed' has a weight equal to, not less than, the static displacement of the vessel, the *Component Waveform Theory* is at odds with conventional design principles. From a 'traditional' viewpoint, the evolution of hulls capable of exceeding 'hull speed' has led to the concept of 'planing' which asserts that, once 'hull speed' has been exceeded, 'dynamic lift' comes into play, somewhat inexplicably, causing a vessel to rise above its 'at rest' position and therefore displace less than its own weight of water. The *Component Waveform Theory* contends, however, that is not actually the case but merely the perception. Although an appropriately shaped vessel travelling at speeds above 'hull speed' is visibly raised above its stationary level, the ball experiment revealed that it does not necessarily follow that the immersed sections of the hull are actually displacing less than the full weight of the vessel. In fact, the *Component Waveform Theory* contends quite the opposite, that for a hull to displace other than the full weight of the vessel is in direct conflict with the principle established by Archimedes.

Straying even further from the modern theoretical approach, rather than adopting the 'flat plate' concept for speeds above 'hull speed', the *Component Waveform Theory* assumes, as it had previously assumed for sub-planing speeds, that, in passing a fixed point at speeds above 'hull speed', each cross-section of a hull, from bow to stern, continues to contribute, in turn, to the displacement of the water particles at that point, combining to generate a component wave in the process. For a vessel to experience a minimum of resistance in generating each component wave at speeds above 'hull speed', some cross-sections of the hull are assumed, therefore, to be pushing water particles aside in some orderly manner while others aft may be matching, perfectly, the return of the water particles towards an equilibrium position. In that regard, the application of the *Component Waveform Theory* for speeds above 'hull speed' remains consistent with that at lower speeds. The theory does recognise, however, that there is a significant change in the circumstances that differentiates planing speeds from sub-planing speeds, the difference being that a vessel moving across the surface of the water at any speed above 'hull speed' is considered to be moving so fast that the water particles displaced by the hull at each point in the vessel's path are still in the process of returning to their equilibrium position after the vessel has passed. How a vessel moving across the surface of the water at a constant speed above 'hull speed' might impact the water particles in its path at any one instant with an infinite number of individual impulses of different magnitudes, delivered by the infinite number of cross-sections along its entire underwater length, is illustrated in Figure 11-2. The *Component Waveform Theory* contends, therefore, that in passing a fixed point at any speed above 'hull speed', a vessel's stern must always be immersed to a depth that matches exactly the situation of the water particles, necessitating an abrupt termination of the hull by means of a transom as initially revealed through the evolution of high-speed powerboats by trial and error in the early decades of the 20th century.

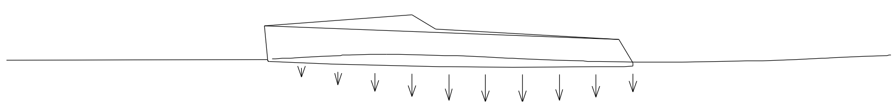


Figure 11-2

Figure 11-3 represents an exaggerated profile of a component wave caused by a full-width disturbance in a parallel-sided channel. The 'disturbance' in this particular case is created by dropping the equivalent of the NS14 sailing dinghy from such a height that the impact triggers the displacement of a volume of water of mass equal to the static displacement of the fully-laden boat but with less of the hull immersed than when stationary.

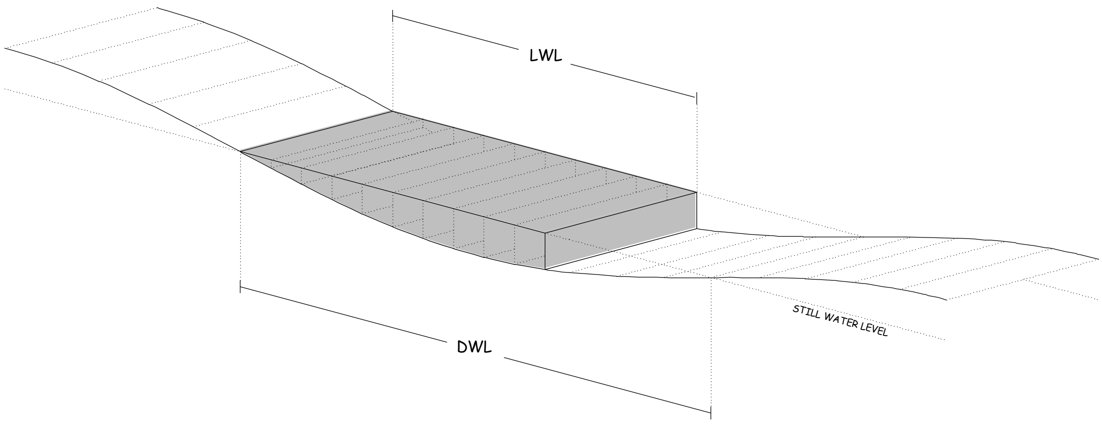


Figure 11-3

The longitudinal asymmetry of the hull creating the 'disturbance' in Figure 11-4 is such that the effect of its impact on the water surface is to create on that water surface a cavity which extends aft beyond the immersed portion of the hull, resulting in a longer wavelength for the component wave than occurred at 'hull speed'.

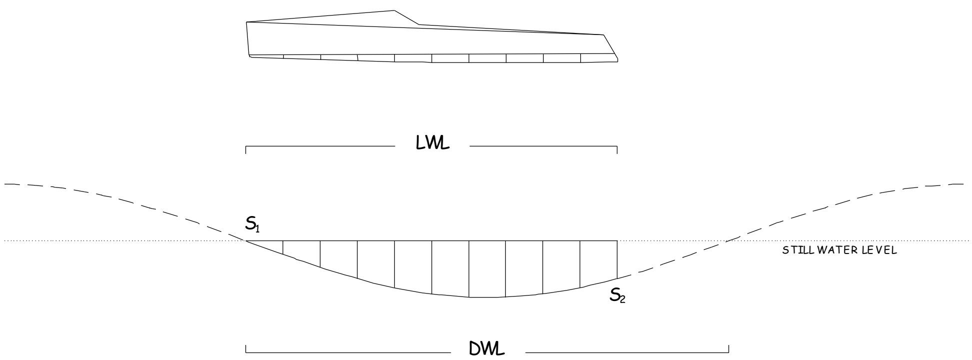


Figure 11-4

S_1 and S_2 in Figure 11-4 are points located at each end of the immersed portion of the hull and the horizontal distance between S_1 and S_2 is the temporary waterline length, shown as LWL. In practice, it is not uncommon for the temporary waterline of a partially immersed hull of a vessel changing trim for planing purposes to be shorter than the static waterline length of the hull. For the sake of consistency in the current discussion, however, in Figure 11-4 the hull's temporary waterline is considered to be equal in length to its static load waterline. To avoid the forward sections of the hull emerging from the water and causing the temporary waterline length to be shortened at speeds above 'hull speed', the boat depicted in the example has been given a plumb stem, the lower part of which is intentionally immersed below still water level at sub-planing speeds. S_1 is located on the surface of the wave at the forward end of the temporary waterline, which is at still water level. S_2, at the aft end of the immersed portion of the hull, is located as shown, on the wave surface below still water level. Obviously, for all speeds higher than 'hull speed', the design waterline length, DWL, which is the effective length over which the hull is actually displacing its own weight of water to create the component wave, is longer than the

LWL. The shaded area bounded by the wave surface, from S_1 to S_2, and the temporary waterline represents the immersed portion of the hull and the location of S_2 indicates an abrupt ending to the hull's immersed aft sections. Clearly, the volume of the immersed portion of the hull is less than the volume of water displaced to form the component wave, which, for all speeds above hull speed, remains equal to the static displacement of the fully-laden boat. Consequently, for all speeds above hull speed the active displacement of the hull is less than its static displacement and the vessel is deemed to be either semi-planing or planing.

During the momentary contact between a hull moving horizontally across the surface of the water and the water particles in its path at any one point, for the circumstance depicted in Figure 11-4 in which the volume of water physically pushed aside is equal to the static displacement of the fully-laden boat, the *Component Waveform Theory* logically assumes, after the ball experiment, that only a portion of the kinetic energy of the boat is transferred to those water particles, setting them in motion. More particularly, the *Component Waveform Theory* assumes that during the impact of the hull on the water particles at each point in the vessel's path at speeds above 'hull speed', only the kinetic energy of the equivalent of that part of the vessel which is below still water level in Figure 11-4 is transferred to the water particles to create the circular component wave. Those water particles are immediately set in motion and, as happened between a speed-length ratio of R=1.7 and 'hull speed', the impulse of the collision is relayed outward from the centre of the disturbance, so that during contact:

$$KE_{hull\ below\ SWL} = (KE + PE)_{displaced\ \ water}$$

Therefore, since the potential energy of the initial wave system is equal to its kinetic energy:

$$KE_{hull\ below\ SWL} = (2KE)_{displaced\ \ water}$$

$$\left(\tfrac{1}{2}mv^2\right)_{hull\ below\ SWL} = 2\left(\tfrac{1}{2}mv^2\right)_{displaced\ \ water}$$

leading to:

$$v^2{}_{vessel} = \frac{2\,(mv^2)_{displaced\ \ water}}{m_{hull\ below\ SWL}}$$

$$v^2{}_{vessel} = 2\,\frac{m_{displaced\ \ water}}{m_{hull\ below\ SWL}}\left(\frac{v_{wave}}{2}\right)^2$$

$$v^2{}_{vessel} = \frac{m_{displaced\ \ water}}{m_{hull\ below\ SWL}}\left(\frac{g\lambda}{4\pi}\right)$$

resulting in:

$$v_{vessel} = \left(\frac{m_{displaced\ \ water}}{m_{hull\ below\ SWL}}\right)^{\frac{1}{2}}\sqrt{\frac{g\frac{\lambda}{2}}{2\pi}}$$

But, at speeds above 'hull speed', the *Component Waveform Theory* has reasoned that the mass of the water displaced during impact to create a component wave at each point in the vessel's path is always equal to the static displacement of the vessel, resulting in the final equation for the velocity of the vessel:

$$v_{vessel} = \left(\frac{m_{hull}}{m_{hull\ below\ SWL}}\right)^{\frac{1}{2}}\sqrt{\frac{g\frac{\lambda}{2}}{2\pi}}$$

Above 'hull speed', therefore, the velocity of a vessel is always greater than the velocity of a wave having half the wavelength of the component wave, as was consistently the case for sub-planing speeds. The determining factor, the ratio of the total mass of the vessel to the equivalent of the mass of the hull which is actually immersed at planing speeds, increases as less of the hull is immersed below still water level at higher velocities.

The effect of the change in the circumstances that differentiates planing speeds from sub-planing speeds is particularly noticeable in the resultant wake. Above 'hull speed' the relationship between the speed of the vessel and the wavelength of the component wave is no longer constant, the faster a surface vessel travels the further the component waves are left behind, so to speak, resulting in the angle of the wake to the direction of the vessel's motion to be visibly reduced, as is the ability of the component waves to reinforce each other in forming the waves within the visible wake. Furthermore, as the speed of a vessel is progressively increased above 'hull speed', the cavity created by the movement of the vessel, as previously illustrated in Figure 11-4, extends further and further aft beyond the immersed portion of the hull, resulting in correspondingly longer wavelengths for the component waves and, because the volume of the water displaced to form each component wave remains constant, equal to the static displacement of the vessel, correspondingly smaller amplitudes. For all of those reasons, the speed at which the waves within a surface vessel's visible wake reach a maximum height is, for an appropriately shaped hull, at the vessel's 'hull speed', beyond which the waves within the wake progressively diminish as the vessel's speed is increased.

From the viewpoint of the *Component Waveform Theory* there appears to be no maximum limit to the speed for which a planing hull can be designed but the effect of the change in the circumstances that differentiates planing speeds from sub-planing speeds is noticeable, not only within the wake but in the behaviour of the vessel itself, particularly within the narrow range of speeds in which a vessel might generally be considered to be 'semi-planing'.

Figure 11-5 illustrates the behaviour of the component waves and the change in trim that an appropriately shaped boat undergoes at a speed marginally above 'hull speed'.

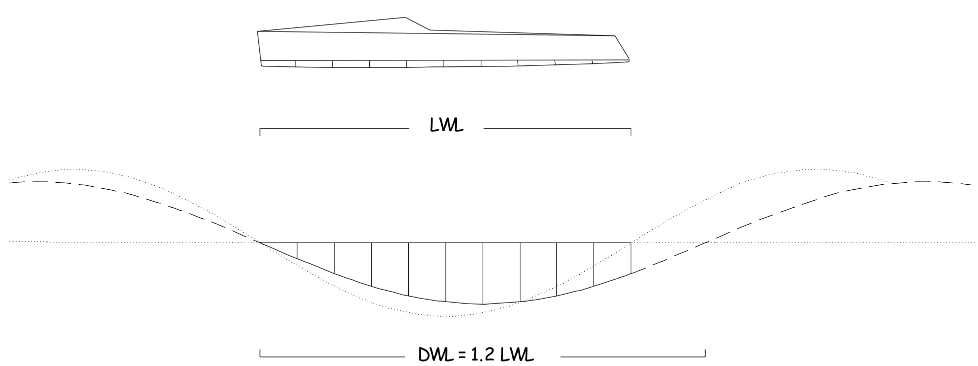


Figure 11-5

As noted previously, the *Component Waveform Theory* contends that as the speed of the boat increases above 'hull speed', so too does the wavelength of the component wave, At the same time, the amplitude of the component wave decreases at a rate inversely proportional to the wavelength so that the volume of water displaced remains constant, equal to the static displacement of the vessel. In Figure 11-5, the design waterline length of the hull, the effective length over which the boat is displacing the water to create each component wave, is equal in length to just 1.2 LWL. A comparison of the immersed cross-sections of the hull at that speed with those when the boat was at 'hull speed' reveals that the forward sections of the hull have risen while the after sections have immersed, the hull seemingly pivoting around the intersection of the two curves, resulting in the familiar 'bow up, stern down' change in trim that is typical of a boat in the very early stages of planing.

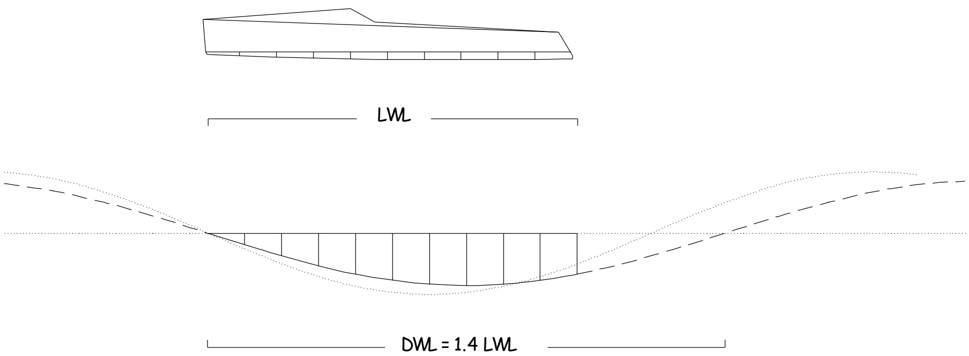


Figure 11-6

Figure 11-6 shows the component wave that is formed at an increased boat speed, where the DWL=1.4 LWL. The supposed pivot point, the intersection of the two curves shown, has moved further aft and the stern is even more submerged than before but, apart from the cross-sections nearest the stern, all of the hull's other cross-sections have risen above their position in Figure 11-5. The hull's 'bow up, stern down' change in trim has become even more pronounced than before and the boat is experiencing the familiar 'hump' as it seemingly struggles to climb and overtake its own bow wave to begin planing.

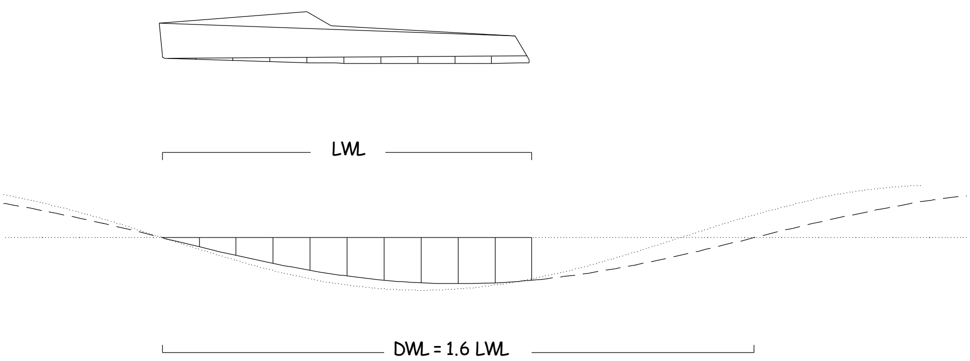


Figure 11-7

By the time the boat has reached a speed where the DWL=1.6 LWL, depicted in Figure 11-7, all of the cross sections forward of the stern have clearly risen and the transom is somewhere near the point of maximum immersion, after which the change of trim will be reversed and the hull will begin to level out, giving the impression to those on board that the bow wave has, at last, been overtaken. Of course, despite that view, in light of the *Component Waveform Theory*, the boat is merely continuing to generate the wake in a similar manner to which it had done at lower speeds but will have, by then, negotiated the semi-planing stage and be entering the next phase in which the hull will be planing fully.

The boat speed at which the maximum immersion of the transom accurs, at some point near the circumstance depicted in Figure 11-7, can be determined more precisely by investigating mathematically the changing shape of the component wave as the speed of the boat is increased.

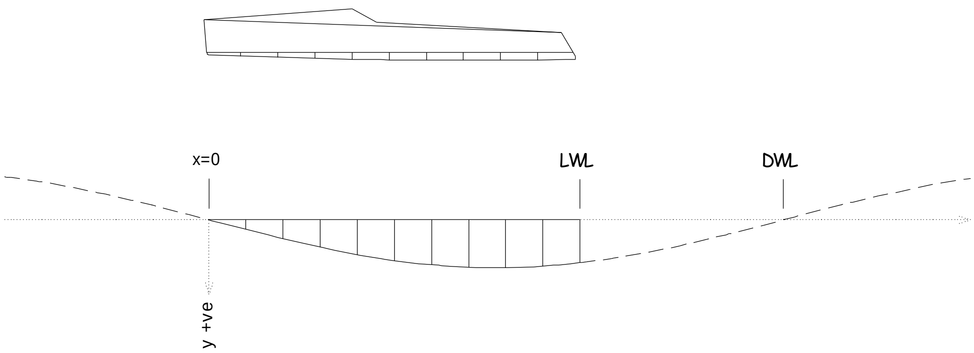


Figure 11-8

Figure 11.8 represents a component wave formed by a hull moving across the surface of the water at an undefined speed above 'hull speed'. By choice, as a first approximation, the function which defines the shape of the wave surface is a sine curve and, for convenience, the location chosen for the origin of the function is at the forward end of the hull's temporary waterline. The y-axis is assumed to be positive in a downward direction.

Consequently, the shape of the wave surface can be given by the general equation:

$$y = a \sin bx + c$$

By inspection, when x=0, y=0 and therefore c=0.

Similarly, when x=DWL, y=0 and therefore b=π/DWL.

Also, when x=DWL/2, by deduction y=a, but to determine the value of 'a' requires further consideration.

The area enclosed by the curve and the x-axis, over the range 0 ≤ x ≤ DWL, represents the static displacement of the boat and is given by the equation:

$$V = \int_0^{DWL} a \sin \frac{\pi x}{DWL} \, dx$$

leading to:

$$V = \frac{a\,DWL}{\pi} \left(-\cos \frac{\pi x}{DWL} \right)_0^{DWL}$$

$$V = \frac{a\,DWL}{\pi}\ (2)$$

which results in:

$$a = \frac{V\pi}{2DWL}$$

Substituting the values of a, b and c into the original equation, the curve approximating the shape of the wave surface becomes:

$$y = \frac{V\pi}{2DWL}\sin\frac{\pi x}{DWL}$$

By definition, V is constant, x and DWL are variable. However, if x is held constant, at x=LWL, it then becomes possible to determine the DWL at which the stern reaches its maximum immersion, the equation for the curve becoming:

$$y = \frac{V\pi}{2DWL}\sin\frac{\pi LWL}{DWL}$$

Differentiating with respect to DWL:

$$\frac{dy}{dDWL} = \frac{V\pi}{2DWL}\cos\frac{\pi LWL}{DWL}\left(-\frac{\pi LWL}{DWL^2}\right) + \sin\frac{\pi LWL}{DWL}\left(-\frac{V\pi}{2DWL^2}\right)$$

leading to:

$$\frac{dy}{dDWL} = \left(-\frac{V\pi}{2DWL^2}\right)\left(\frac{\pi LWL}{DWL}\cos\frac{\pi LWL}{DWL} + \sin\frac{\pi LWL}{DWL}\right)$$

which equals zero if the DWL is infinite or when:

$$\left(\frac{\pi LWL}{DWL}\cos\frac{\pi LWL}{DWL} + \sin\frac{\pi LWL}{DWL}\right) = 0$$

leading to:

$$\left(\frac{\pi LWL}{DWL} + \tan\frac{\pi LWL}{DWL}\right) = 0$$

$$\left(\tan\frac{\pi LWL}{DWL}\right) = -\frac{\pi LWL}{DWL}$$

and finally, by trial and error:

$$\frac{\pi LWL}{DWL} = 2.029$$

resulting in:

$$DWL = 1.548\,LWL$$

In other words, using the sine curve approximation for the shape of the component wave surface, calculations show that the stern reaches its maximum immersion when the speed of a vessel is such that the DWL is equal to approximately 1.55 LWL.

The volume of the immersed portion of the hull when full planing commences, at a boat speed where the DWL is equal to 1.55 LWL, can also be determined, being the area enclosed by the curve and the x-axis over the the range 0 ≤ x ≤ LWL, the length of the temporary waterline, and given by the equation:

$$V_{hull\ imme\ rsed} = \int_0^{LWL} \frac{V\pi}{2DWL} \sin\frac{\pi x}{DWL} dx$$

where V is the static displacement of the vessel.

Integrating:

$$V_{\ hull\ immersed} = \frac{V\pi}{2DWL} \frac{\text{DWL}}{\pi} \left(-\cos\frac{\pi x}{DWL}\right)_0^{\text{LWL}}$$

leading to:

$$V_{hull\ immersed} = \frac{V}{2}\left(1 - cos\frac{\pi LWL}{DWL}\right)$$

But, from the previous calculation, for the speed at which a boat begins to fully plane the DWL is equal to 1.548 LWL, allowing the ratio LWL/DWL to be evaluated in the equation above, so that ultimately:

$$V_{hull\ immersed} = 0.72V$$

That is, when a vessel has reached that part of the semi-planing phase at which the stern has reached its maximum depth of immersion and the hull is beginning to plane fully, the volume of hull that is actually immersed is approximately equal to 72% of the static displacement of the vessel.

Investigating further, the relative velocity at which the stern reaches its maximum depth of immersion and a hull begins to fully plane can now be resolved as well.

Previously, the equation for the velocity of a vessel travelling across the surface of the water at speeds above 'hull speed' was determined to be:

$$v_{\ vessel} = \left(\frac{m_{\ hull}}{m_{\ hull\ below\ \ SWL}}\right)^{\frac{1}{2}} \sqrt{\frac{g\frac{\lambda}{2}}{2\pi}}$$

Substituting the known values for the speed at which a hull begins to fully plane:

$$v_{\ vessel} = \left(\frac{100}{72}\right)^{\frac{1}{2}} \sqrt{\frac{g(1.548LWL)}{2\pi}}$$

which leads finally to:

$$v_{\ vessel} = (1.47)\sqrt{\frac{gLWL}{2\pi}}$$

The velocity at which the stern of an appropriately shaped vessel reaches its maximum depth of immersion and its hull begins to fully plane is, therefore, determined by the *Component Waveform Theory* to be at approximately 1.47 times the vessel's velocity at 'hull speed' or, in other words, at a relative velocity of about R=3.6.

Interestingly, the *Component Wavefore Theory* appears to be generally in agreement with what is considered by observation to be the speed at which a hull begins to plane, but the 'flat plate' concept seems not to apply until a higher speed is reached at which the hull's design waterline length is at least twice the length of the hull's load waterline, as depicted in Figure 11-9, a speed which happens to be at a relative velocity of approximately R=4.9, twice that at 'hull speed' and at which the volume of the immersed portion of the hull is equal to 50% of the static displacement of the vessel.

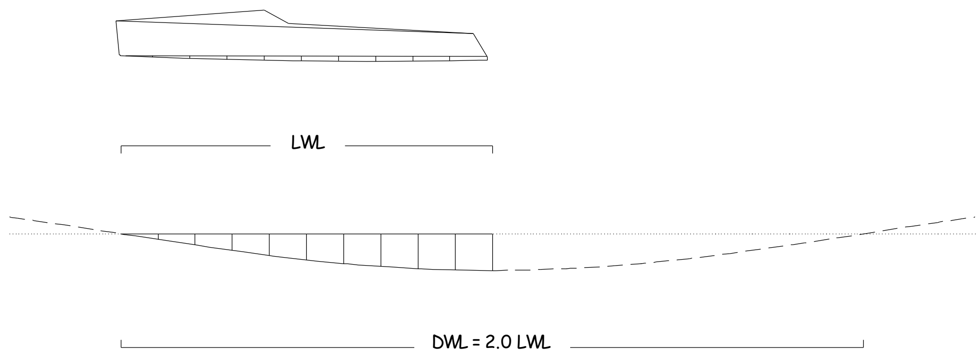
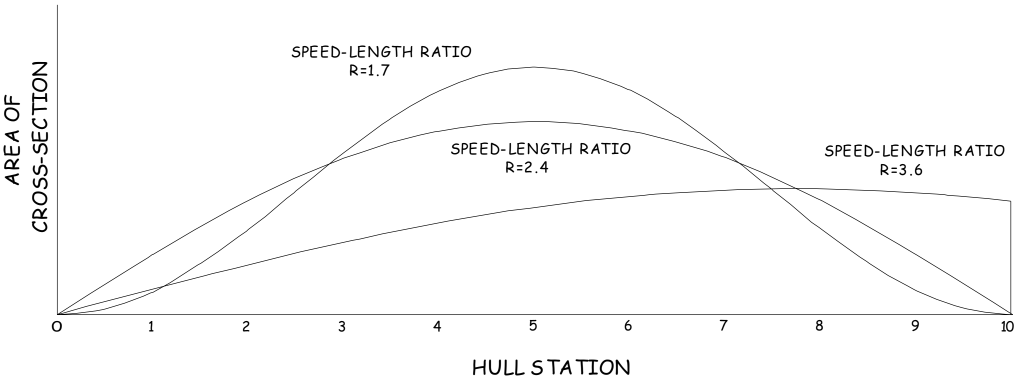


Figure 11-9

Although full planing could be considered to have begun at about R=3.6, Figure 11-8, which does, in fact, depict the hull at the speed at which the stern is immersed to its greatest depth, shows clearly that there are cross-sections of greater area forward of the transom. Not until a speed is reached at which the hull's design waterline length is twice the length of the hull's load waterline, as in Figure 11-9, does the immersed stern actually become the cross-section of greatest area, which remains the case at all higher speeds. Only at very high speeds, as the component wave lengthens and flattens, does the area curve begin to resemble that of a 'flat plate'.

The conventional approach to the design of planing hulls, adopting the 'flat-plate' concept to enable the calculation of the hydrodynamic forces acting on a hull, evolved with the emergence of high-speed powerboats in the early decades of the 20^{th} century and is still used by designers for that purpose. Application of the *Component Waveform Theory* is, in contrast, more specific and, for that reason alone, is particularly suited to sailboat design in that the theory allows the generation of a precise curve of areas for any planing speed, including those speeds within the critical low-speed range of a sailboat's semi-planing phase, marginally above the 'hull speed' barrier.

Figure 11-10 depicts some of the variations possible in the shapes of the area curves derived using the *Component Waveform Theory.* At a speed-length ratio of R=1.7 the design waterline length is at a 'minimum' and the vessel is in displacement mode, at R=2.4 the vessel has reached 'hull speed' while R=3.6 marks the end of the semi-planing phase, the vessel's stern is at its maximum depth of immersion, all sections of the hull are rising and the volume of the vessel's hull that is actually immersed is little more than about 70% of the static displacement of the vessel.



SPEED-LENGTH RATIO
R=1.7
SPEED-LENGTH RATIO
R=2.4
SPEED-LENGTH RATIO
R=3.6
AREA OF
CROSS-SECTION
0
1
2
3
4
5
6
7
8
9
10
HULL STATION

Figure 11-10

THE THEORY APPLIED

In application, the fundamental difference between the *Component Waveform Theory* and conventional design principles is one of approach. Aided by increasingly sophisticated technology, the trend for designers of all types of vessels since the closing decades of the 20th century has been to employ ever more complex hydrodynamic formulae, supported by the accumulating results of extensive tank-testing of scale models throughout the world's research institutions, to predict, as accurately as possible, the total interaction between a proposed design and the elements under realistic sailing conditions. Over time, that trend has inevitably resulted in the routine application of design processes about which the designer may actually know little or nothing and, not surprisingly, the modern design process, for designers of all types of vessels, is gradually becoming more and more remote. In contrast, the *Component Waveform Theory* is reminiscent of a much earlier period in the history of recreational sailboat design, merely electing to isolate and virtually 'freeze' just one aspect of a hull's performance to enable the use of elementary mathematical and scientific reasoning so that a designer might not only acquire a broad understanding of the interaction between a hull and the water at various speeds under ideal conditions but, as a consequence, be provided with a logical method by which the longitudinal displacement of hull shapes of least wave-making resistance can be roughly determined.

The *Component Waveform Theory* is particularly suited to the design of small planing sailboats which have developed over the past century, mostly conceived by amateur designers using trial and error techniques that typically have spawned very little accurate performance data for future designers to exploit. To demonstrate the basic principles of how the *Component Waveform Theory* can be applied to such hull types, following is an outline of the procedures that enable an estimation of the optimum longitudinal displacement of hull shapes of least wave-making resistance for velocities above a speed-length ratio of about R=1.7, by application of the theory.

Hull Speed

For a vessel moving across the surface of the water at 'hull speed', the *Component Waveform Theory* assumes that the amount of kinetic energy continually being transferred to the water particles is such that the volume of water pushed aside in the formation of a single component wave at each point in the vessel's path is equal to the static displacement of the vessel.

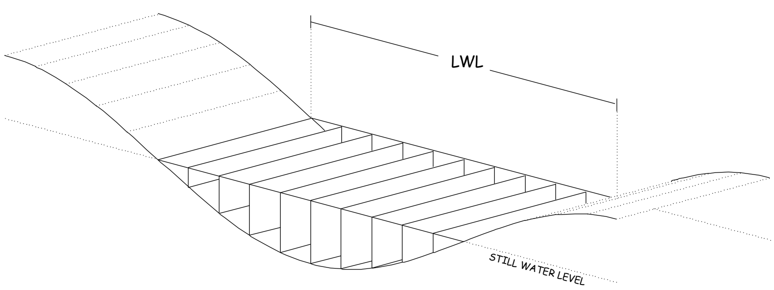


Figure 12-1

Figure 12-1 shows, in three dimensions, the component wave that was created in Figure 9-1 by dropping the equivalent of an NS14 hull onto the water surface within a parallel-sided channel from such a height that the impact triggered the displacement of a volume of water equal in weight to the static displacement of the fully laden boat.

In Figure 9-2, each vertical component of the curve below still water level was assumed to represent a cross-sectional area of the underwater portion of the NS14's hull, disregarding the actual shape of the cross-section. Clearly, from Figure 12-1, the area of any such cross-section is equal to the product of the immersed depth of the equivalent hull at that point and the width of the channel into which that hull was dropped. Likewise, the volume of water displaced by the NS14's equivalent hull is directly related to the amplitude of the component wave and is equal to the product of the area bounded by the curve, below still water level, and, once again, the width of the channel. By assuming a suitable width for the channel, therefore, and scaling the sine curve so that the volume of water displaced is equal in weight to that of the fully laden NS14, the cross-sectional areas of the underwater portion of the NS14's hull at 'hull speed' can be determined directly, either graphically, mathematically or by a combination of both methods.

Displacement Speeds

For a vessel moving across the surface of the water at any speed below 'hull speed', the *Component Waveform Theory* assumes that only a portion of the total kinetic energy of the vessel is transferred to the water particles with which it collides at every point in its path, forming a component wave and setting in motion a volume of water of mass less than the static displacement of the vessel at each of those points.

For speeds between R=1.7 and 'hull speed', the range of displacement speeds over which the *Component Waveform Theory* can be applied, the vessel's hull is reasoned to be partially raised above still water level by the formation of the component wave. Consequently, for speeds between R=1.7 and 'hull speed', the hull's design waterline length at still water level is always shorter than the load waterline length and, for simplicity, rather than the selection of a preferred design speed, the selection of an appropriate design waterline length becomes a logical starting point for determining the dimensions of the component wave.

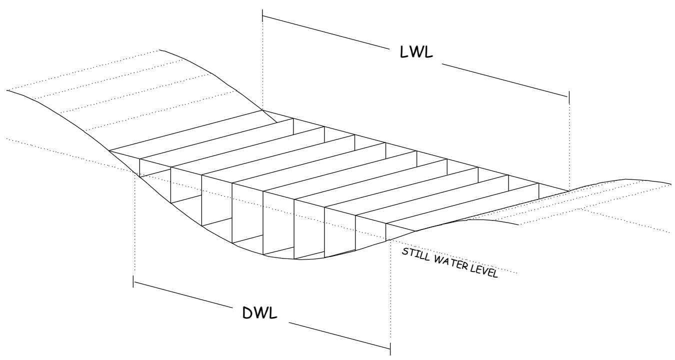


Figure 12-2

Figure 12-2 shows, in three dimensions, the component wave that was created in Figure 10-1 by dropping the equivalent of an NS14 hull onto the water surface within the parallel-sided channel from such a height that the impact triggered the displacement of a volume of water of mass less than the static displacement of the fully laden boat.

As at 'hull speed', the volume of water displaced by the NS14's equivalent hull in Figure 12-2 is directly related to the amplitude of the component wave and the width of the channel. At all displacement speeds between R=1.7 and 'hull speed', however, the static displacement of the boat is represented by the volume of the equivalent hull below the load waterline, which is located above still water level. The area of any cross-section of the NS14's equivalent hull, depicted in Figure 12-2, is equal, therefore, to the product of the depth of the hull below the load waterline at that cross-section and, as before, the width of the channel. By assuming the width of the channel and scaling a sine curve similar to that depicted in Figure 10-2 so that the volume of the equivalent hull below the load waterline is equal to the static displacement of the fully-laden NS14, the cross-sectional areas of the underwater portion of the NS14's hull having the nominated design waterline length can be determined directly.

Similarly, the volume of water actually displaced by the NS14's hull to form the component wave at that particular speed is the volume of the equivalent hull below the design waterline, which is located at still water level.

Semi-planing and Planing Speeds

For a vessel moving across the surface of the water at any speed above 'hull speed', the *Component Waveform Theory* assumes that only a portion of the total kinetic energy of the vessel is transferred to the water particles with which it collides at every point in its path, forming a component wave and setting in motion a volume of water of mass equal to the static displacement of the vessel at each of those points.

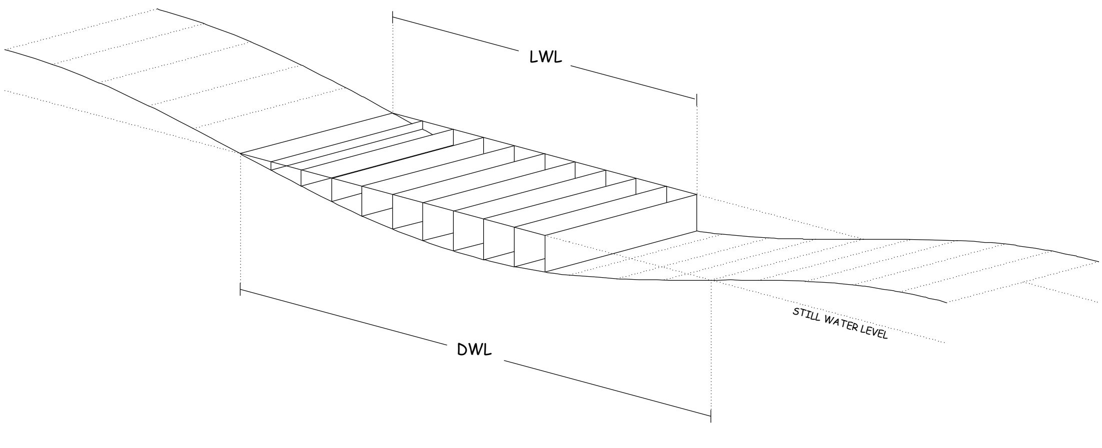


Figure 12-3

Figure 12-3 shows, in three dimensions, the component wave that was created in Figure 11-3 by dropping the equivalent of an NS14 hull onto the water surface within the parallel-sided channel from such a height that the impact triggered the displacement of a volume of water of weight equal to the static displacement of the fully laden boat but with less of the hull immersed than when stationary. As a matter of interest, the hull depicted in Figure 11-3 was previously shown to be imparting the same amount of energy to the water as a hull travelling across the water surface at the speed at which the transom is naturally immersed to its maximum depth and the hull is beginning to plane fully.

For all speeds above 'hull speed', the hull's design waterline length is inevitably longer than the load waterline length and, as for displacement speeds, rather than the selection of a preferred design speed, the selection of an appropriate design waterline length becomes, once again, a logical starting point for determining the dimensions of the component wave.

As at all other speeds above R=1.7, the volume of water displaced by the NS14's equivalent hull in Figure 12-3 is directly related to the amplitude of the component wave and the width of the channel. For all speeds above 'hull speed', however, whether semi-planing or planing fully, the static displacement of the boat is represented by the volume of water displaced from below still water level over the length of the design waterline to form the component wave. The area of any cross-section of the NS14's equivalent hull, depicted in Figure 12-3, is equal, therefore, to the product of the depth of the hull below still water level at that cross-section and, as before, the width of the channel. By assuming the width of the channel and scaling a sine curve similar to that depicted in Figure 11-4 so that the volume of water displaced from below still water level over the length of the design waterline is equal in weight to that of the fully-laden NS14, the cross-sectional areas of the underwater portion of the NS14's hull having the nominated design waterline length can be determined directly.

Similarly, the volume of the NS14's hull actually immersed to form the component wave at that particular speed is the volume of that portion of the equivalent hull below the temporary load waterline at still water level.

The Trochoidal Waveform

The *Component Waveform Theory* is based on the assumption that during the formation of the circular wave pattern caused by a point disturbance in open water, the effect of the impulse moving outward from the centre of the disturbance is comparable to the transverse wave pattern caused by a full-width disturbance of the same magnitude in a parallel-sided channel of appropriate width. Adopting a perfect, non-viscous, incompressible, elastic fluid of infinite depth and extent, as Isaac Newton had assumed for his theory of resistance and František Gerstner for the derivation of his *Trochoidal Wave Theory*, the profile of the resulting transverse wave pattern is accepted as being trochoidal, and so the curve representing the cross-sectional areas of the hull is also assumed to be trochoidal. To demonstrate the basic principles of the theory, however, the trochoid has, so far, been substituted by the much simpler sine curve, which is a very close approximation to the trochoid when the amplitude of the curve is very small in comparison to the wavelength.

In practice, the key advantages of using the sine curve as a curve of areas in preference to a trochoidal waveform are that the sine curve is easily evaluated and no matter to what extent the curve of areas is scaled, horizontally or vertically, the proportional relationship between the various cross-sections of the hull remains unchanged, allowing the length and amplitude of a common curve of areas to be adjusted directly to match any hull's design waterline length and displacement.

Trochoidal curves, on the other hand, are difficult to evaluate, being defined by the equations:

$$x = R\theta - r\sin\theta$$

$$y = r - r\cos\theta$$

Unlike the sine curve, the value of y on a trochoidal curve cannot be expressed as a direct function of x, being linked, instead, through three other variables, R, r and θ. As a consequence, altering just one component of the trochoid, such as the value of R, for example, which determines the wavelength of the curve, ultimately changes the actual shape of the curve as well. Simply scaling a common trochoidal curve of areas to match a nominated design waterline length and displacement is, therefore, out of the question. Instead, the curve of areas for each of the nominated design waterline lengths of a hull must be calculated individually, which is not a straightforward undertaking.

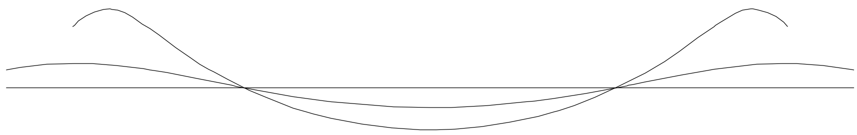


Figure 12-4

For comparison, Figure 12-4 shows just two of the variety of shapes possible for the trochoidal component waves, each having the same design waterline length. Clearly, not only are the shapes and wavelengths of the trochoidal curves in Figure 12-4 different, the areas representing the volume of water displaced are also markedly different, suggesting that for a nominated design waterline length and displacement, the actual shape of the trochoidal wave is directly dependent on the selected width of the channel. Dropping the equivalent of an NS14 hull into a wide channel, for example, would produce a shallow trochoidal wave, while an equivalent hull of the same design waterline length and displacement, dropped into a narrow channel, would produce a steeper, higher wave of shorter wavelength, as depicted in Figure 12-4. Consequently, unlike the sinusoidal waveform, for which a suitable width of the channel could be chosen at will, for the trochoidal waveform selection of the correct width of the channel is crucial in determining the true shape of the component wave and, as a consequence, the true shape of the related curve of areas. Ultimately, only by linking the energy of the trochoidal waveform produced in the channel with that of the energy input of the hull, can the channel's exact width be determined.

From Gerstner's *Trochoidal Wave Theory*, the energy of one wavelength of a trochoidal wave can be expressed as:

$$Energy_{\,wave} = \frac{1}{2}\rho g \lambda r^2 \left(1 - \frac{r^2}{2R^2}\right) \; per\ unit\ breadth\ of\ wave$$

If the width of the channel is, in fact, the 'unit breadth of wave' referred to in the equation above, the width of the channel becomes, in effect, the 'apparent beam', ß, of the NS14's equivalent hull. Calculation of ß from the respective energy equations requires repeated estimations of the configuration of the trochoidal curve until the dimensions that match the intended design waterline length and displacement of the hull are determined.

Fortunately, in practice, the cross-sectional areas obtained by estimating the curve of areas of a hull by using a sinusoidal profile for the component wave vary by only a very small percentage from those obtained by using the trochoidal profile. For that reason alone, given the extremely variable conditions under which sailboats are forced to operate, use of the more demanding trochoidal waveform to determine a more precise curve of areas could be considered unwarranted.

Figure 12-5 compares the two chosen profiles for the trochoidal component waves with sine curves of equal design waterline length. Clearly, above still water level the trochoidal curves differ markedly from the sine curves, but below still water level the two sine curves are almost identical in shape to the trochoids. The wavelengths of the trochoidal curves are noticeably shorter, which means that the velocity calculations using results obtained by adopting a sine curve distribution of a hull's longitudinal displacement are an approximation only, impacting on the energy input calculations as well. However, the more specific use of the sine curve to determine solely the curve of areas of the immersed portion of a hull of known design waterline length and displacement is a practical and satisfactory alternative, particularly for vessels optimised for 'hull speed' or beyond.

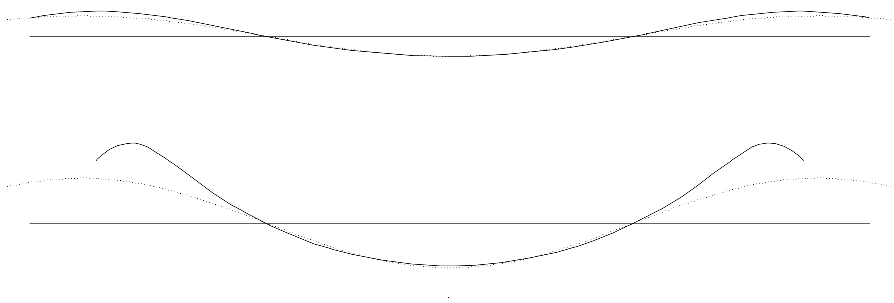


Figure 12-5

In practice, after selecting an appropriate design waterline length based on an estimate of the speed for which a vessel of known load waterline length and displacement is being designed, application of the *Component Waveform Theory* to determine the optimum curve of areas of the hull by using the sinusoidal profile for the component wave is not only reasonably accurate but also surprisingly uncomplicated. For simplicity, therefore, the trochoidal waveform is substituted hereafter by the much simpler sine curve.

The Cross-sectional Areas

In Figures 12-1 to 12-3, the dimensions of the component waves and the hull's cross-sections are exaggerated for clarity. To demonstrate the application of the *Component Waveform Theory* more fully, it is appropriate that the theory be applied to an actual hull design, the choice of which is, once again, an NS14, a conventional monohull development class sailing dinghy which allows variations in design within some simple measurement restrictions.

Relevant to the application of the *Component Waveform Theory,* the principal restriction of the NS14 class of dinghy is the overall length of the hull, which is limited to between 4.25 and 4.30 metres, giving a maximum load waterline length of, say, 4.25 metres. For the purpose at hand, an NS14 in full sailing trim with a crew of two, is assumed to have a total weight of 225 kilograms and intended for use in salt water, accepted as having a density of approximately 1024 kilograms per cubic metre. The volume of water displaced by the hull at rest, in fresh water with a density of 1000 kilograms per cubic metre, would be 0.225 cubic metres, or 225 litres. In salt water, on the other hand, the volume of water displaced by the hull at rest is reduced to approximately 0.220 cubic metres, or 220 litres.

Various results obtained for the NS14 sailing dinghy by application of the *Component Waveform Theory* are presented in the appendix, *A Design for a Dinghy*, covering a range of speeds from R=1.7, through 'hull speed' and the semi-planing phase to about R=8.0, at which point the hull is planing at relatively high speed. Cross-sectional areas of the NS14's hull for a selection of nominated design waterline lengths are shown and are approximations abtained by application of the *Component Waveform Theory* using graphical methods only, substituting the trochoidal waveform with the much simpler sine curve.

By appropriately scaling the cross-sectional areas displayed in *A Design for a Dinghy,* the optimum cross-sectional areas of any hull having a load waterline length and displacement different from those of the NS14 sailing dinghy used in the example, can be determined directly for each nominated ratio of design waterline length to load waterline length.

The Volume of the Hull Below Still Water Level

Recognition and evaluation of the volume of a hull below still water level at any given speed, as well as the mass of an equivalent volume of water, are fundamental to the application of the *Component Waveform Theory.*

Results of calculations of the volume of the NS14's hull below still water level, as well as the mass of an equivalent volume of water, are presented for a range of design waterline lengths in *A Design for a Dinghy*. Figure A1-3 presents the results in graphical form.

For speeds below 'hull speed', the curve presented in Figure A1-3 represents the volume of water that is displaced to form each component wave. When in displacement mode, the immersed volume of the NS14's hull is considered to be always equal to the static displacement of the boat. However, as the boat proceeds across the water surface at displacement speeds, at least above a speed-length ratio of R=1.7, the *Component Waveform Theory* contends that the NS14's hull is raised above its 'at rest' position by the component wave that is created and that only the volume of the hull below still water level represents the lesser volume of water displaced at each point in the boat's path to form the component wave.

As can be seen from Figure A1-3, above the comfortable 'cruising' speed of R=1.7, the volume of water displaced to form each component wave is reasoned by the *Component Waveform Theory* to increase significantly as the boat's speed approaches 'hull speed', at which point the volume of water displaced to form the component wave is considered to be exactly equal to the static displacement of the boat.

For speeds above above 'hull speed', however, the circumstances reverse.

When the NS14 is in semi-planing and planing modes, the volume of water displaced to form each component wave is reasoned by the *Component Waveform Theory* to always remain equal to the static displacement of the boat but, as clearly evident in Figure A1-3, the volume of the NS14's hull below still water level gradually decreases as the boat's speed increases, with the obvious consequence that, at all speeds above 'hull speed', the NS14 actually has less of its hull in the water than when at rest.

Calculation of the volume of the hull below still water level is, according to the *Component Waveform Theory,* essential for determining the velocity of the hull when semi-planing or planing fully, for determining the kinetic energy imparted to the water by the hull at any speed to form each component wave and, in turn, for determining the least possible resistance to forward motion experienced by the hull in creating each of those waves.

The Hull's Input of Kinetic Energy to Create the Component Wave

Having determined the volume of the NS14's hull below still water level for each of the nominated design waterline lengths as well as the mass of an equivalent volume of water, the boat's velocity that corresponds to each of the nominated design waterline lengths can then be calculated, leading, in turn, to the calculation of the kinetic energy imparted to the water by the NS14's hull to create the corresponding component wave.

Results of calculations of the NS14's input of kinetic energy to create the component waves over a range of speeds are presented in *A Design for a Dinghy*. Figure A1-4 presents those same results in graphical form and also shows the relationship between the NS14's input of kinetic energy with the boat's total kinetic energy.

The kinetic energy of any object in motion is equal to the work it can do before it is brought to rest. From the graph in Figure A1-4, the amount of excess energy available to the NS14 hull at all speeds, except 'hull speed', is clearly evident and explains, then, why a hull travelling at slow speeds or at very high speeds, can continue for some distance without propulsion at almost the same speed, while a hull travelling near 'hull speed' loses way almost immediately. On the water, that effect is particularly noticeable in the case of a small, fast powerboat which for some reason suddenly loses power. At first, the powerboat hull continues to plane normally for a short distance but, as the boat slows, it also gradually sinks lower into the water until, abruptly, 'hull speed' is reached, at which point the hull appears to almost stop and then momentarily surge forward to continue very slowly in displacement mode before finally coming to rest. Clearly evident in Figure A1-4, the high proportion of a hull's kinetic energy required to form the component waves near 'hull speed' creates a very real barrier to forward motion, particularly for sailboats, for which the necessary propulsive force is not always easily maintained.

For displacement speeds approaching 'hull speed', the curve presented in Figure A1-4 rises sharply, an obvious indication of the extra force required to propel any craft at speeds above the comfortable 'cruising speed' of R=1.7. At 'hull speed', however, there is a distinct break in the shape of the curve, marking the end of the displacement phase and the beginning of the semi-planing phase. Beyond 'hull speed', the curve continues to rise but at a much lesser rate, indicating that any increase in power beyond 'hull speed' results in a greater increase in speed for a favourably shaped hull than could occur at high displacement speeds. The shape of the curve in Figure A1-4 beyond 'hull speed' also suggests that, in theory at least, there is no limit to the speed at which a boat can travel.

The Resistance Due to the Formation of Waves

From the outset, the intention of the *Component Waveform Theory* was solely to match the longitudinal displacement of a vessel's hull to that of the component waves being formed at any chosen design speed so as to minimise the interaction between the hull and the water particles without the need to determine the actual magnitude of the forces involved. However, from the application of the *Component Waveform Theory* to the design of the NS14 used as an example, it becomes evident that having first determined the optimum cross-sectional areas of the NS14's hull for each of the nominated design waterline lengths and then having established the boat's velocity for each of those design waterline lengths, the volume of water displaced by the movement of the boat to form a component wave at every point in the boat's path at each of those velocities and the input of kinetic energy necessary to form the respective component waves, the means by which to calculate the minimum wave-making resistance of the NS14 at any chosen design speed is within reach.

Traditionally, the wave-making resistance of a hull has generally been regarded, by laymen at least, as a vague and obscure retarding force, or 'drag', which inhibits the forward motion of a vessel as it moves across the surface of the water. From that historic perspective, calculation of the magnitude of the wave-making resistance of a vessel at different speeds has long been considered as an almost unattainable goal. Adopting the viewpoint of the *Component Waveform Theory,* however, the wave-making resistance of a hull is, on the whole, not considered to be a retarding force, or 'drag', at all but, more precisely, a measure of the force which a hull necessarily has to apply to the water particles in the direction of its travel to push the water particles aside in forming each component wave. Unnecessary 'drag' due to the formation of the wake is, according to the *Component Waveform Theory*, only experienced when the hull's longitudinal displacement is in conflict with that of the component waves naturally being formed.

During the collision between a hull and the water particles at every point in a vessel's path, energy is conserved but, as determined previously, in forming the component waves some of the vessel's kinetic energy is transferred via the hull to the water particles. Clearly evident in Figure A1-4, the actual proportion of a vessel's kinetic energy that is transferred to the water particles is variable and dependent on the relative speed of the vessel. In the process of transferring that energy, a force is applied to the water particles by the hull over a very short time interval and in a very complex manner that, in reality, cannot be accurately defined. To overcome that problem, the *Component Waveform Theory* has assumed, from the very beginning, that as a hull passes a fixed point on the water surface, the impulsive force applied to the water particles by the hull to create a single component wave, acts over the short length of time the hull takes to go by and varies in strength throughout the collision in direct proportion to the immersed areas of the hull's passing cross-sections, as depicted in Figure 8-6. The loss of kinetic energy that a vessel experiences in passing each point on the water surface is deemed, therefore, to be a measure of the cumulative effect of the sequential forces applied to the water particles by the hull's cross-sections and is, in turn, a measure of sorts of the overall force which represents the hull's so-called resistance to forward motion due to the formation of waves. Interestingly, because the component waves are formed as a result of the collisions between a hull and the water particles in its path, any estimation of a hull's so-called resistance to forward motion due to the formation of waves by use of the *Component Waveform Theory* is likely to include that portion of a hull's resistance which was discovered by Mark Beaufoy in the late 18th century and subsequently attributed to friction, even for the smoothest of hulls.

For a vessel at 'hull speed', the *Component Waveform Theory* contends that the total amount of work done by the impulsive force at each point in the vessel's path to form just one component wave is equal to the total kinetic energy of the vessel. If, as in Figure 8-6, the sum of the individual impulses impacting the water particles as each cross section passes a fixed point at 'hull speed' is equal to the total amount of work done to form each component wave, then the average force being applied to the water particles by the hull at each point in the vessel's path to generate a single component wave at 'hull speed' can be given by the equation:

$$F = \left(\frac{KE}{LWL}\right)_{vessel}$$

leading to:

$$F = \left(\frac{\frac{1}{2}mv^2}{LWL}\right)_{vessel}$$

$$F = \left(\frac{\frac{1}{2}m\left(g\frac{\lambda}{2}/2\pi\right)}{\frac{\lambda}{2}}\right)_{vessel}$$

$$F = \left(\frac{mg}{4\pi}\right)_{vessel}$$

During the momentary contact between the hull moving across the water surface at 'hull speed' and the water particles in its path, the entire kinetic energy of the vessel is, therefore, transferred to the water particles by the impulsive force calculated above, setting the particles in motion. In open water, however, the impulse of the collision radiates outward from the centre of the disturbance, equally in all directions. In so doing, only one-quarter of the energy of that initial impulse radiates outward in any given direction. Logically, then, the force exerted by the hull on the water particles in the direction in which the vessel is travelling is only one-quarter of the force of that initial impulse, giving the result:

$$F = \left(\frac{mg}{16\pi}\right)_{vessel}$$

The force exerted by the hull on the water particles in the direction in which the vessel is travelling is generally referred to as the vessel's resistance to forward motion due to the formation of waves or, more simply, as the vessel's wave-making resistance. In practical terms, employing the usual system of measurement and expressing the calculated force in kilograms rather than Newtons, the force exerted by the hull on the water particles in the direction in which the vessel is travelling at 'hull speed' can, therefore, be expressed as:

$$F_{resistance} = \frac{m_{\ vessel}}{16\pi}$$

Immediately, some remarkable aspects of a vessel's wave-making resistance at 'hull speed' emerge. To begin with, the magnitude of the force involved in creating the wake of a vessel at 'hull speed', according to the derived formula, seems surprisingly low, being only about 2% of the overall displacement of the vessel, approximately 4.5 kilograms in the case of the NS14 used as an example. Yet, despite the typical perceptions of those on board of the considerable power required to drive any sailboat at 'hull speed', such a result compares favourably with recorded measurements. Also of interest is that the formula's lone variable which determines the magnitude of the wave-making resistance of a vessel at 'hull speed', regardless of the load waterline length of the hull, is the vessel's static displacement. In other words, according to the formula, all appropriately shaped vessels of the same weight, whatever their load waterline length, experience the same wave-making resistance at their individual 'hull speed', another unexpected outcome for most. Perhaps even more interesting, though, is that the formula derived to calculate the wave-making resistance of a vessel at 'hull speed' makes no provision whatsoever for the shape of the hull, giving an impression that weight, not hull shape, is all important to the performance of a vessel and that the power-weight ratio of a vessel is what truly determines any vessel's ultimate speed potential, an impression easily disproven, however, by the simple experiment of towing a spherical toy balloon horizontally across the water surface for probably the most unexpected outcome of all.

For speeds other than 'hull speed', the forces exerted by a hull on the water particles in the direction in which a vessel is travelling are applied in a similar manner to that of a vessel at 'hull speed' but over the different length of time that the hull takes to pass a fixed point on the water surface. Clearly, a component wave of set dimensions can be formed in any number of ways. The wave created by a planing hull, for example, is identical to that created by a hull of the same static displacement but having a load waterline length equal to the planing hull's design waterline length and travelling at its individual 'hull speed'. As each of those two hulls passes a fixed point on the water surface an equal amount of energy is imparted to the water particles to create the identical component waves, the significant difference being the length of time over which the impulsive force is applied. Importantly, though, during the collision between a hull and the water particles in its path, not only is energy conserved but so too is the momentum of the masses involved in the collision. Consequently, if the planing hull and the hull at 'hull speed' are to produce identical component waves, as suggested, then the momentum of those respective masses prior to the collisions must be identical also. Similar logic applies to the case of a vessel moving across the surface of the water at displacement speeds above R=1.7.

In brief, if F_1 represents the impulsive force applied by the hull to the water particles at 'hull speed' and Δt_1 is the time taken for that hull to pass a fixed point on the water surface, then by adopting the principle of the conservation of momentum, the force, F_2, applied by the hull at speeds other than 'hull speed', over the interval Δt_2, can be determined from the equation:

$$F_1 \Delta t_1 = F_2 \Delta t_2$$

$$F_2 = \frac{F_1 \Delta t_1}{\Delta t_2}$$

$$F_2 = \frac{\left(\frac{m_{displaced\ \ water}}{16\pi}\right)\left(\frac{DWL}{\sqrt{g\frac{\lambda}{2}/2\pi}}\right)}{\left(\frac{LWL}{v_{hull}}\right)}$$

At planing speeds the force exerted by a hull on the water particles in the direction in which the vessel is travelling can, therefore, be expressed as:

$$F_{resistance} = \left(\frac{m_{hull}}{16\pi}\right)\left(\frac{DWL}{LWL}\right)\left(\frac{m_{hull}}{m_{hull\ below\ \ SWL}}\right)^{\frac{1}{2}}$$

For displacement speeds the formula reduces to:

$$F_{resistance} = \left(\frac{m_{displaced\ \ water}}{16\pi}\right)\left(\frac{DWL}{LWL}\right)$$

Estimations of the minimum resistance to the forward motion of an NS14 hull, designed by application of the *Component Waveform Theory,* are presented for a range of design waterline lengths in *A Design for a Dinghy*. Figure A1-5 presents those same results in graphical form and also shows, for comparison, a typical 'humped' resistance curve of a powerboat of similar dimensions to the NS14 but designed to the flat-plate principle and intended for operation at a speed well beyond the potential of an NS14 under sail.

As a matter of interest, the formula derived for the minimum wave-making resistance of a hull at any speed can be expressed generally as a function of the input of kinetic energy and the design waterline length:

$$F_{resistance} \propto \left(\frac{KE_{input}}{DWL}\right)$$

leading to:

$$F_{resistance} \propto Av^2$$

which, if A represents the average cross-sectional area of the component wave below still water level, is fundamentally consistent with the formula derived by Isaac Newton in the 17th century.

The Design

Although an approximation, application of the *Component Waveform Theory* using the sinusoidal waveform provides the designer with a wealth of information. Above all, of course, is the computation of the optimum cross-sectional areas of a hull of known displacement for a range of design waterline lengths. Further application of the theory offers a reasonably accurate estimation of the hull's speed and wave-making resistance for each of those design waterline lengths, suitable for comparison rather than for any other purpose, allowing a hull shape of minimum wave-making resistance to be created for any chosen design speed.

Various results obtained for the NS14 sailing dinghy by application of the *Component Waveform Theory* , adopting a sinusoidal waveform for the component wave, are presented in the appendix, *A Design for a Dinghy*, covering a range of speeds from R=1.7, through 'hull speed' and the semi-planing phase to about R=8.0, at which point the hull is planing at relatively high speed.

As always, the challenge for the designer is the selection of the appropriate data to develop a hull form to suit the intended use of the boat.

For sailing in a particular location, for example, the designer might deduce that typical weather conditions would allow an NS14 to be sailed upwind at 'hull speed' more often than not but off the wind, with a relatively small area of working sail and without a spinnaker, the expected planing speeds would usually be low, if planing occurred at all. Local experience on the water may have shown, however, that NS14 hulls capable of planing efficiently off the wind in marginal conditions had generally proven to be more successful. The range of boat speeds for which the NS14 hull would need to be honed for such conditions might seem, therefore, to be between 'hull speed' and, perhaps, the speed at which the hull begins to fully plane, in other words, the semi-planing phase. In terms of design waterline lengths, from the data presented in *A Design for a Dinghy,* that particular speed range for an NS14 lies between the design waterline lengths of 4.25 metres and 6.58 metres. The dilemma confronting the designer, however, is that a hull can be shaped to minimise wave-making resistance for one speed only, all other speeds creating unnecessary 'drag' due to the longitudinal displacement of the hull being in conflict with that of the component waves being formed. Inevitably, even for the narrow range of design speeds selected, the shape of the hull must necessarily be compromised.

If an emphasis is to be placed on a boat's planing ability off the wind in marginal weather conditions, a reasonable choice for the optimum design waterline length of the NS14's hull might be 6.58 metres, the design waterline length at which the hull's transom reaches its maximum immersion and the hull begins to plane fully. Obviously, the NS14's hull could be configured for that design waterline length alone, distributing the calculated underwater volume of the hull in a shape that minimises the wetted surface area to reduce the effects of friction. Figure A1-6 in the appendix, *A Design for a Dinghy*, shows a preliminary investigative design for such an NS14, achieved without reference to pre-existing hull shapes by utilising the sectional areas calculated for DWL 6580 alone to create circular cross-sections below the design waterline. The necessity to cater for the anticipated slower speed upwind, however, raises the question of whether a secondary design waterline length could possibly be incorporated into the design.

Figure 12-6 shows the NS14's area curves for both 'hull speed' and the upper end of the boat's semi-planing phase, the speed at which the hull begins to plane fully.

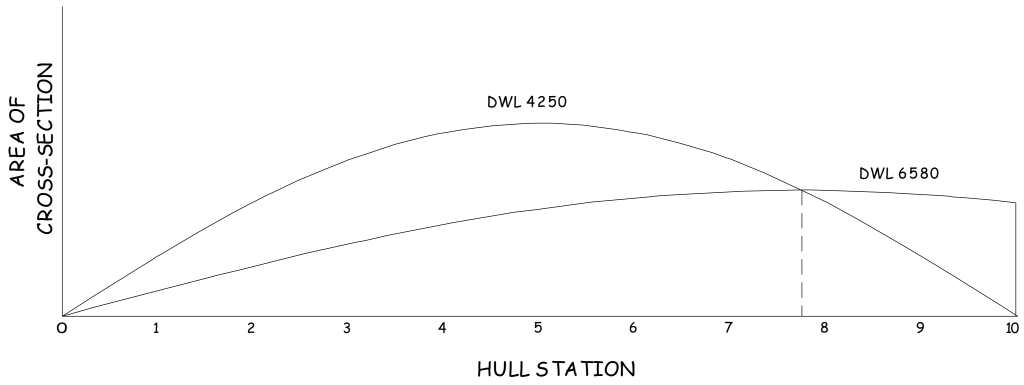


Figure 12-6

The intersection of the two area curves, between stations 7 and 8, represents an immersed cross-section of the NS14's hull that is common to both design waterline lengths. Consequently, in moving instantaneously from 'hull speed' to fully planing, or vice versa, the hull could be considered to have rotated around that common cross-section, altering the hull's longitudinal trim in the process, as depicted in Figure 12-7.

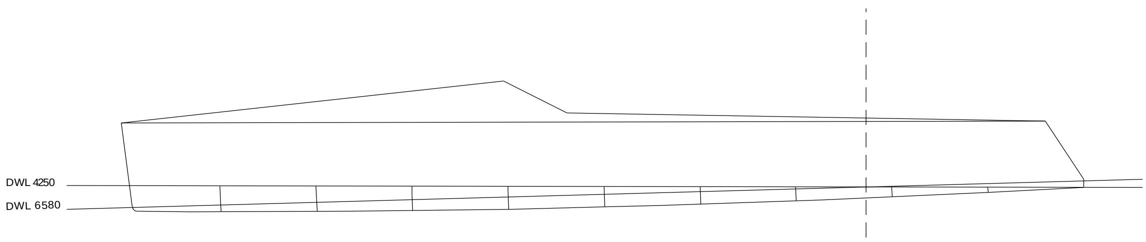


Figure 12-7

After further deliberation it becomes apparent that by slightly modifying whatever is considered to be the optimised shape for planing at DWL 6580, the cross-sectional areas necessary to minimise the wave-making resistance at 'hull speed', or any other speed, might also be absorbed into the design to some extent, with the prospect of the designer being able to generate, mathematically, a compromised shape for the hull from scratch. The reality, though, is that there is no limit to the number of options available to the designer to 'perfect' the design of the sailboat for the given set of circumstances.

Essentially, the concept of being able to achieve a uniquely shaped hull of least wave-making resistance for any surface vessel is unsound. In practice, no matter how beneficial an understanding of the longitudinal displacement a hull moving across the surface of the water at any particular speed may seem, calculation of the appropriate 'curve of areas' is but one element of a complex multifaceted design process in which, needless to say, the skill and the experience of the designer must ultimately prevail. Be that as it may, after selection of the displacement, the load waterline length and the design waterline length of an intended design, application of the *Component Waveform Theory* to calculate the optimum 'curve of areas' does, at the very least, provide a unique start-point from which the actual shapes of a hull's cross-sections can be judiciously developed to satisfy a mixture of design criteria without adversely affecting the hull's ability to proceed across the surface of the water with a minimum of resistance to its forward motion brought about by the creation of the wake.

FACT OR FICTION?

Sailboats are unique and, despite millennia of evolution, their perfection continues to challenge designers and academics alike.

For more than five thousand years before the Age of Steam, the sailing ship evolved at the forefront of human achievement and, understandably, throughout that period shipbuilding was largely based on traditional techniques, relying heavily on the experience of individual designer-builders to interpret the performance of existing vessels and to devise appropriate improvements for new designs. Not until the late 17th century, in the midst of an international scientific revolution, was the possibility of the existence of a uniquely shaped 'solid of least resistance' first proposed, with Isaac Newton expressing the uncomplicated notion that the resistance to forward motion experienced by any object moving through a fluid is likely the direct result of the object's impact with the fluid particles in its path. Experiments undertaken by Newton led to his proposal that an immersed object's resistance to forward motion is proportional to the square of its velocity and the maximum cross-sectional area which the object presents to the fluid. When applied by others to ships moving, instead, across the surface of a fluid, Newton's conclusions proved to be incompatible with practical experience. As a consequence, for the three hundred years or more since, Newton's elemental concept of resistance has been practically abandoned by academia, replaced by a swell of increasingly complex theories that are yet to provide a comprehensive alternative.

Rightly or wrongly, in an attempt to devise a means by which the resistance of a vessel moving across the surface of the water might be minimised and in contrast to the accepted principles of design, the *Component Waveform Theory* adopts a similar approach to that taken by Newton for an immersed object but, in addition, makes some allowance for the surface waves that are inevitably formed in the process. By adopting Newton's truly fundamental approach, the *Component Waveform Theory* avoids the complexities brought about by subsequent analyses of the flow of fluids and, in particular, the introduction of streamlines. Moreover, unlike John Scott Russell's *Waveline Theory* or Colin Archer's *Waveform Theory,* and in contrast to the typical approach to the question of wave-making resistance, the *Component Waveform Theory* is not actually dependent on a detailed analysis of the visible wake. Yet, despite its simplicity, the *Component Waveform Theory* manages to provide a reasonable and consistent explanation for the behaviour of surface vessels as they progress from displacement speeds through the 'hull speed' barrier to semi-planing and planing speeds. On the whole, despite the *Component Waveform Theory's* unconventional and relatively crude methodology, the specific results obtained by application of the theory to the one class of surface vessel used for demonstration purposes do appear to be in general agreement with measured observations.

Application of the *Component Waveform Theory* to sailboat design has shown that the 'hull speed' barrier is not a myth but a very real obstacle to the progress of any conventional surface vessel, in much the same way that the 'sound barrier' impedes the progress of aircraft, raising the likelihood of a similar approach to that of the *Component Waveform Theory* to determine the shapes of least resistance for objects that are, in fact, fully immersed. Looking further afield, application of the *Component Waveform Theory* to the disturbance created on the surface of the oceans by the gravitational effect of the moon can provide an extremely simple and logical explanation of how two lunar tides are produced simultaneously on opposite sides of the earth, likening the disturbance to that produced by a surface vessel at displacement speeds, a disturbance which has been shown to leave in its wake transverse waves having wavelengths equal to approximately half that of the component wave created at every point in the vessel's path.

All things considered, in gauging the success or otherwise of the *Component Waveform Theory* in achieving a practical contribution to the sailboat design process, however limited, the one haunting doubt that naturally persists is whether or not the theory, as described, does adequately portray what actually occurs when a vessel proceeds across the surface of the water. From a purely academic perspective, the unavoidable conclusion must be that although the *Component Waveform Theory* seemingly produces acceptable outcomes, the theory is, in fact, little more than an undemanding approach to sailboat design that largely misinterprets or completely ignores the realities of fluid behaviour. Regardless of the predictability and the certainty of such an assessment, from a layman's perspective the *Component Waveform Theory*, whatever its faults, does, at the very least, offer a rare and constructive insight into some otherwise extremely vague aspects of sailboat performance and of wave-making, the principal barrier to high speed under sail.

APPENDIX

A DESIGN FOR A DINGHY

The following tables show the optimum longitudinal displacement of a hull at various velocities above a speed-length ratio of R=1.7, as determined by application of the *Component Waveform Theory.*

The hull type chosen on which to demonstrate the application of the *Component Waveform Theory* is an NS14, a conventional monohull development class sailing dinghy which allows variations in design within some simple measurement restrictions. The principal restriction of the NS14 class of dinghy, relevant to the application of the *Component Waveform Theory,* is the overall length of the hull, which is limited to between 4.25 and 4.30 metres, giving a maximum load waterline length of, say, 4.25 metres. Additional measurement restrictions apply to the shape of the cross-section located 2.5 metres from the forward end of the load waterline.

For the purpose at hand, the NS14 is intended for use in salt water, accepted as having a density of approximately 1024 kg/m^3. An NS14 in full sailing trim with a crew of two, is assumed to have a total weight of 225 kilograms, the hull at rest therefore displacing approximately 0.220 m^3, or 220 litres of salt water.

The abbreviations and units used throughout the following tables are :

LWL (mm) :	Load waterline length of the hull (millimetres).
DWL (mm) :	Design waterline length of the hull, equal in length to the portion of the component wave below still water level (millimetres).
λ (m) :	Wavelength of the component wave (metres).
V_{SWL} (litre) :	Volume of the portion of the hull below still water level (litres).
m_{SWL} (kg) :	Mass equivalent of the portion of the hull below still water level (kilograms).
$V_{water\ displaced}$ (litre) :	Volume of water displaced to form each component wave (litres).
$m_{water\ displaced}$ (kg) :	Mass of water displaced to form each component wave (kilograms).
v_{hull} (m/sec) :	Velocity of the hull (metres per second).
v_{hull} (knot) :	Velocity of the hull (knots).
R (knot, m) :	Speed-length ratio (knots, metres).
KE Input (joule) :	Kinetic energy of the mass equivalent of the portion of the hull below still water level (joules).
Resistance (kg) :	Minimum resistance to the forward motion of the hull (kilograms).

To demonstrate the basic principles of the *Component Waveform Theory,* the trochoidal waveform has been substituted by the much simpler sine curve and that curve has been evaluated by graphical means only. By appropriately scaling the cross-sectional areas displayed in *A Design for a Dinghy,* the optimum cross-sectional areas of any hull having a load waterline length and displacement different from those of the NS14 sailing dinghy used in the example, can be determined directly for each nominated ratio of design waterline length to load waterline length.

Figure A1-1 illustrates the link between the immersed sections of the hull and the changing shape of the component wave for the range of boat speeds in the tables that follow, the hatched sections representing the hull's optimum 'curve of areas' for each of those speeds.

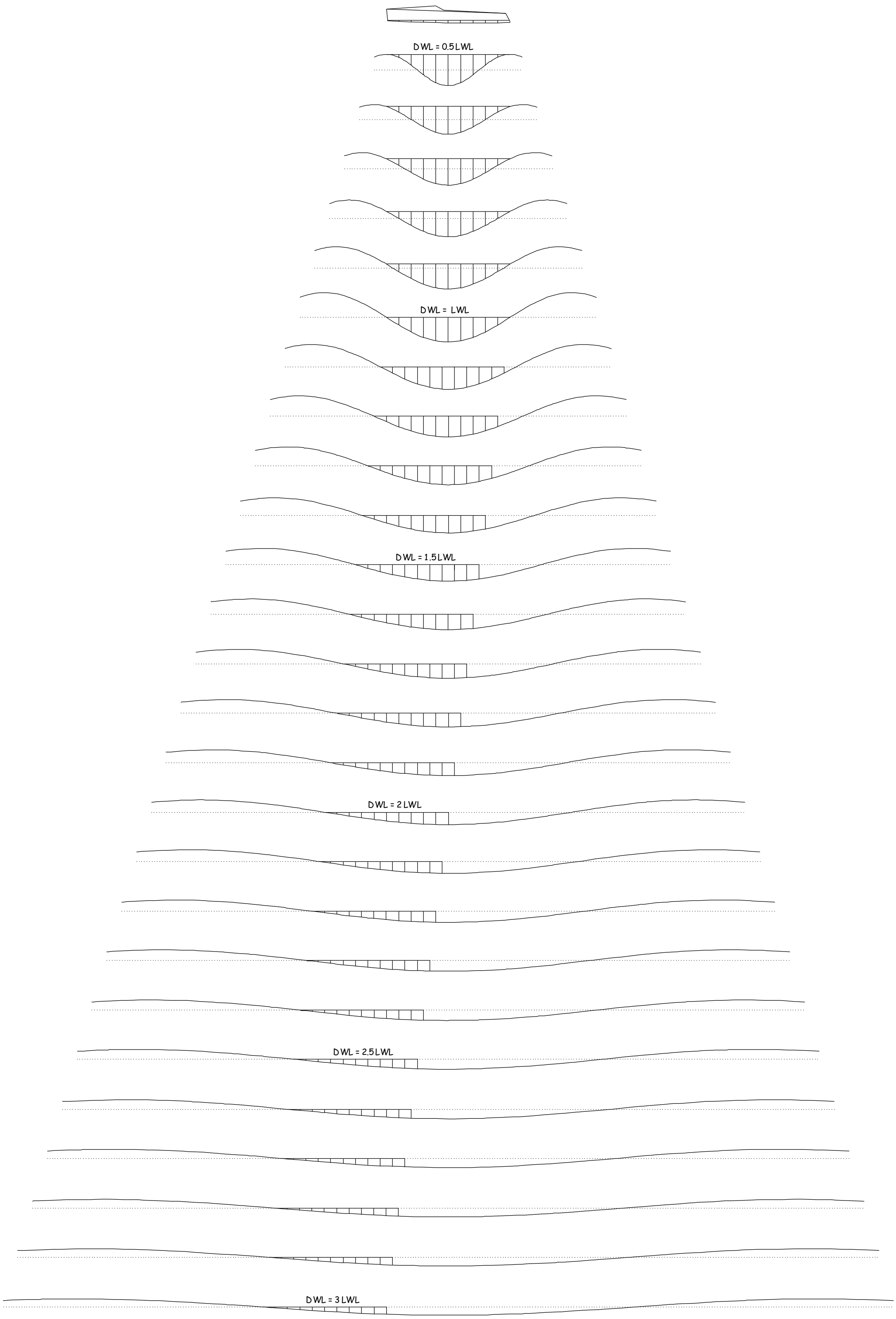


Figure A1-1

NS14 – 225S : AREAS OF CROSS-SECTIONS FOR DISPLACEMENT SPEEDS (cm^2)

DWL/LWL :	0.5	0.6	0.7	0.8	0.9	1.0
DWL (mm) :	2125	2550	2975	3400	3825	4250
STN						
0	0	0	0	0	0	0
425	99	179	213	232	243	251
850	357	424	450	464	472	477
1275	677	668	663	660	658	657
1700	935	847	811	792	780	772
2125	1034	913	864	838	822	812
2500	957	861	822	802	789	781
2550	935	847	811	792	780	772
2975	677	668	663	660	658	657
3400	357	424	450	464	472	477
3825	99	179	213	232	243	251
4250	0	0	0	0	0	0
λ (m) :	4.25	5.10	5.95	6.80	7.65	8.50
V_{SWL} (litre) :	70	79	101	131	171	220
m_{SWL} (kg) :	72	81	103	134	175	225
$V_{water\ displaced}$ (litre) :	70	79	101	131	171	220
$m_{water\ displaced}$ (kg) :	72	81	103	134	175	225
v_{hull} (m/sec) :	1.8	2.0	2.2	2.3	2.4	2.6
v_{hullL} (knot) :	3.5	3.9	4.2	4.5	4.7	5.0
R (knot,m) :	1.72	1.88	2.03	2.17	2.30	2.43
KE Input (joule) :	119	162	239	356	521	746
Resistance (kg) :	0.7	1.0	1.4	2.1	3.1	4.5

NS14 - 225S : AREAS OF CROSS-SECTIONS FOR SEMI-PLANING SPEEDS (cm^2)

DWL/LWL :	1.1	1.2	1.3	1.4	1.5	1.55
DWL (mm) :	4675	5100	5525	5950	6375	6580
STN						
0	0	0	0	0	0	0
425	208	175	150	129	113	106
850	399	338	290	252	220	207
1275	558	479	414	362	318	300
1700	672	586	514	454	402	381
2125	731	654	584	523	469	446
2500	734	676	618	562	511	488
2550	731	677	620	565	515	492
2975	672	654	620	580	538	519
3400	558	586	584	565	538	524
3825	399	479	514	523	515	508
4250	208	338	414	454	469	470
λ (m) :	9.35	10.20	11.05	11.90	12.75	13.16
V_{SWL} (litre) :	215	205	192	178	165	159
m_{SWL} (kg) :	220	210	197	183	169	162
$V_{water\ displaced}$ (litre) :	220	220	220	220	220	220
$m_{water\ displaced}$ (kg) :	225	225	225	225	225	225
v_{hull} (m/sec) :	2.7	2.9	3.1	3.4	3.6	3.8
v_{hullL} (knot) :	5.3	5.7	6.1	6.6	7.1	7.3
R (knot,m) :	2.57	2.75	2.96	3.19	3.43	3.55
KE Input (joule) :	820	895	969	1044	1119	1154
Resistance (kg) :	5.0	5.6	6.2	7.0	7.8	8.2

NS14 - 225S : AREAS OF CROSS-SECTIONS FOR LOW PLANING SPEEDS (cm^2)

DWL/LWL :	1.6	1.7	1.8	1.9	2.0
DWL (mm) :	6800	7225	7650	8075	8500
STN					
0	0	0	0	0	0
425	99	88	78	70	64
850	194	173	154	139	125
1275	282	251	226	203	184
1700	359	322	290	263	239
2125	422	381	346	315	287
2500	464	423	386	353	324
2550	469	428	391	358	328
2975	498	460	424	391	362
3400	508	476	444	414	386
3825	498	476	451	426	401
4250	469	460	444	426	406
λ (m) :	13.60	14.45	15.30	16.15	17.00
V_{SWL} (litre) :	152	140	129	119	110
m_{SWL} (kg) :	156	143	132	122	113
$V_{water\ displaced}$ (litre) :	220	220	220	220	220
$m_{water\ displaced}$ (kg) :	225	225	225	225	225
v_{hull} (m/sec) :	3.9	4.2	4.5	4.8	5.1
v_{hullL} (knot) :	7.6	8.2	8.8	9.4	10.0
R (knot,m) :	3.69	3.96	4.25	4.55	4.85
KE Input (joule) :	1193	1268	1342	1417	1491
Resistance (kg) :	8.6	9.5	10.5	11.6	12.7

NS14 - 225S : AREAS OF CROSS-SECTIONS FOR HIGHER PLANING SPEEDS (cm^2)

DWL/LWL :	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
DWL (mm) :	8925	9350	9775	10200	10625	11050	11475	11900	12325	12750
STN										
0	0	0	0	0	0	0	0	0	0	0
425	58	53	48	44	41	38	35	32	30	28
850	114	104	95	88	81	75	69	65	60	56
1275	168	153	141	129	120	111	103	96	89	84
1700	218	200	183	169	157	145	135	126	118	110
2125	263	242	223	206	191	177	165	154	144	135
2500	298	275	254	236	219	204	190	178	167	156
2550	302	279	258	239	222	207	193	181	169	159
2975	335	311	288	268	250	234	219	205	193	181
3400	360	336	314	293	274	257	241	227	213	201
3825	377	354	333	313	294	277	260	246	232	219
4250	386	365	346	327	309	292	276	261	247	234
λ (m) :	17.85	18.70	19.55	20.40	21.25	22.10	22.95	23.80	24.65	25.50
V_{SWL} (litre) :	102	94	87	81	76	71	66	62	58	55
m_{SWL} (kg) :	104	96	90	83	78	73	68	64	60	56
$V_{water\ displaced}$ (litre) :	220	220	220	220	220	220	220	220	220	220
$m_{water\ displaced}$ (kg) :	225	225	225	225	225	225	225	225	225	225
v_{hull} (m/sec) :	5.5	5.8	6.2	6.6	6.9	7.3	7.7	8.1	8.5	8.9
v_{hullL} (knot) :	10.7	11.3	12.0	12.7	13.5	14.2	15.0	15.7	16.5	17.3
R (knot,m) :	5.17	5.50	5.83	6.18	6.53	6.89	7.26	7.63	8.02	8.41
KE Input (joule) :	1566	1641	1715	1790	1864	1939	2013	2088	2163	2237
Resistance (kg) :	13.8	15.0	16.3	17.7	19.0	20.5	22.0	23.6	25.2	26.9

NS14 - 225S : OPTIMUM CURVE OF AREAS AT CRITICAL BOAT SPEEDS

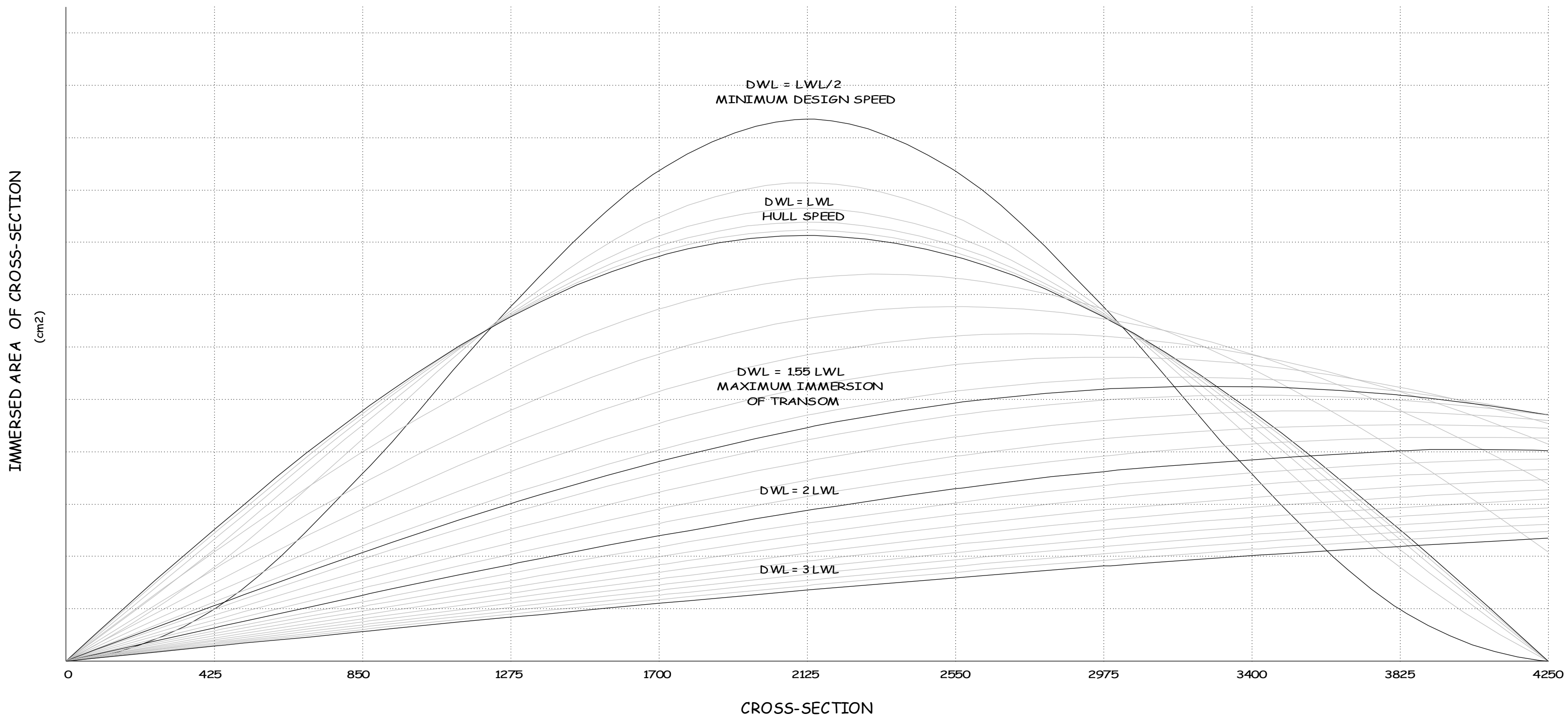


Figure A1-2

Note that the curve of areas for the boat speed at which the DWL is equal to three times the length of the LWL is almost a straight line, closely resembling the curve of areas of a flat plate, traditionally used to explain the mechanics of 'planing' at higher speeds.

NS14 - 225S : VOLUME OF HULL BELOW STILL WATER LEVEL

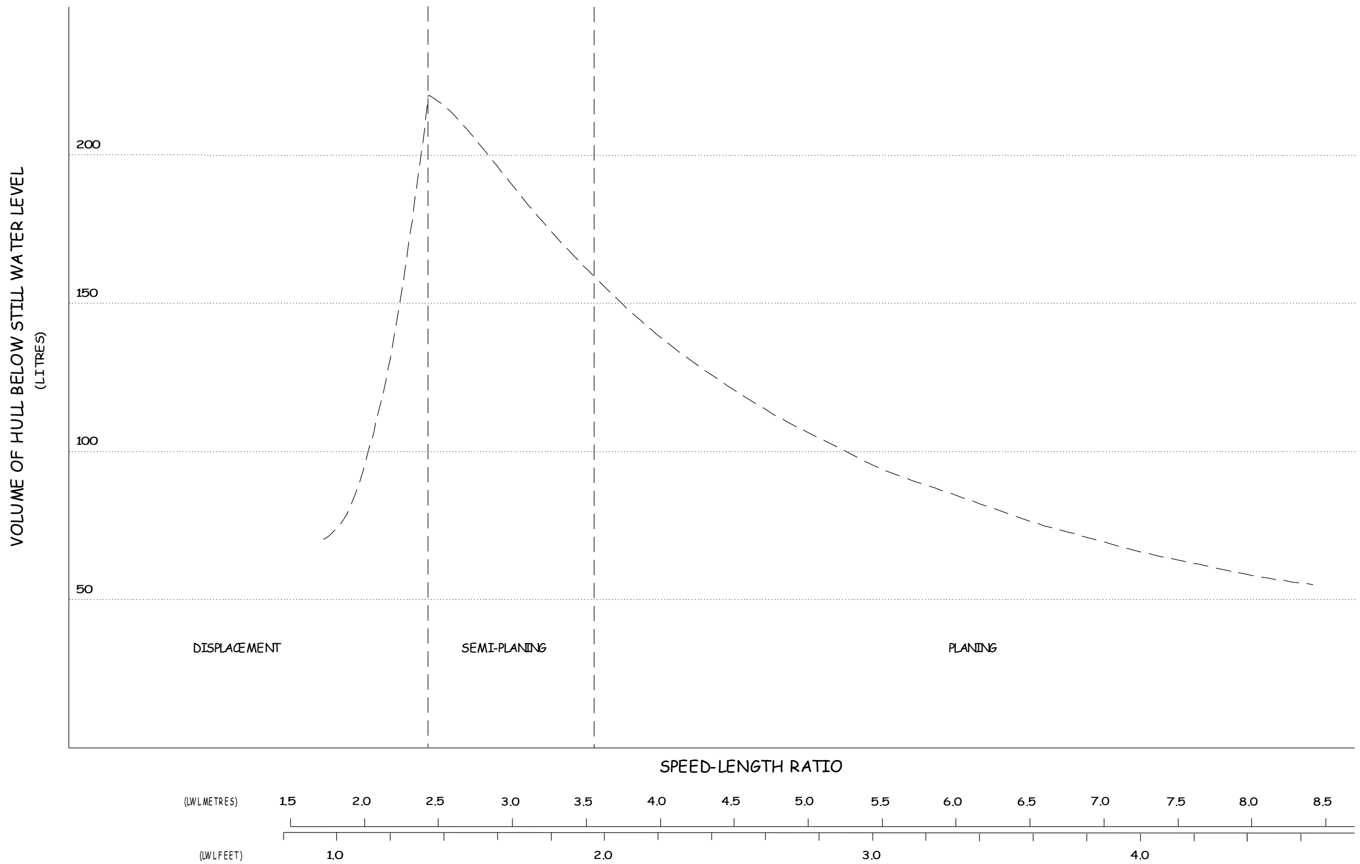


Figure A1-3

NS14 - 225S : KINETIC ENERGY INPUT TO CREATE EACH COMPONENT WAVE

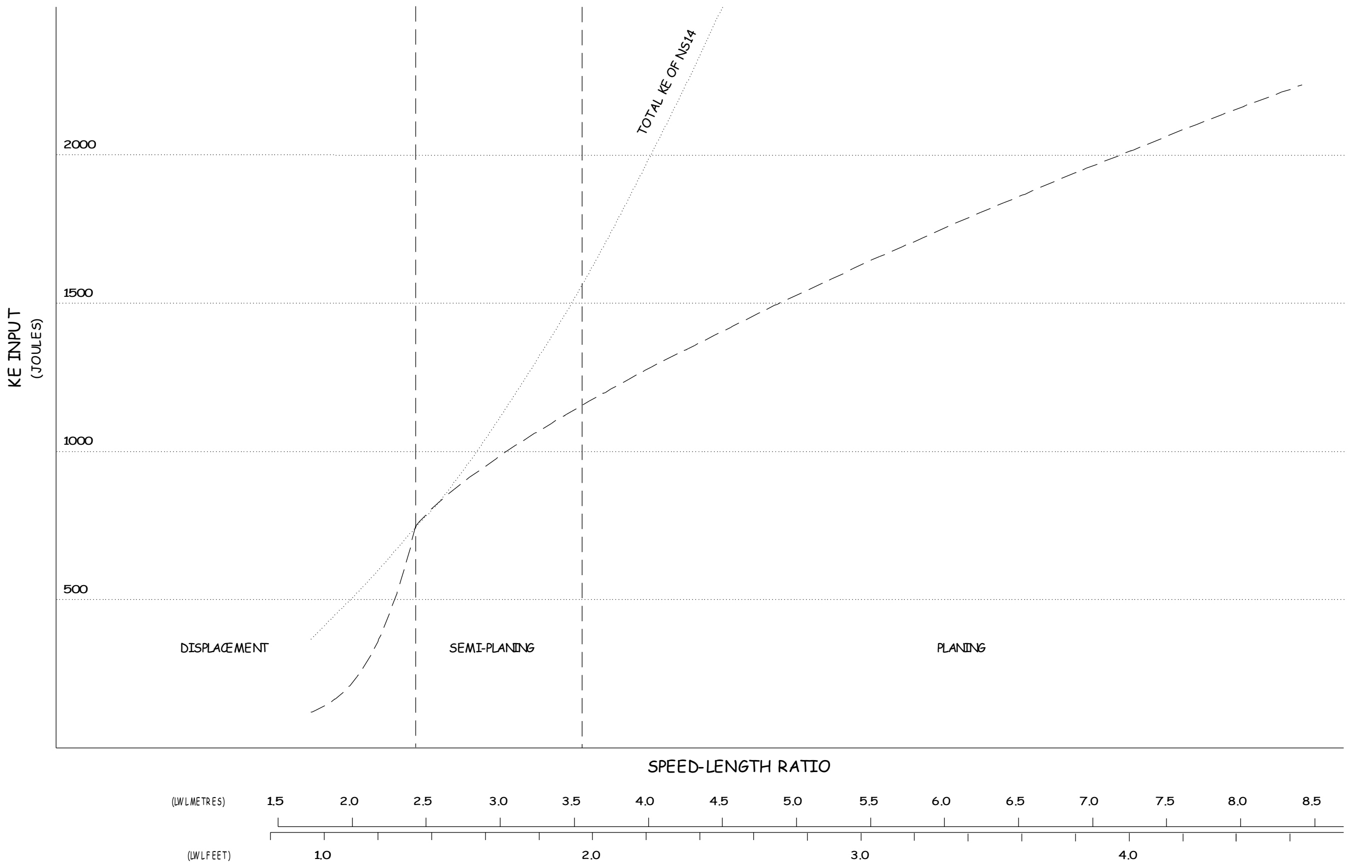


Figure A1-4

NS14 - 225S : MINIMUM RESISTANCE DUE TO THE FORWARD MOTION OF THE HULL

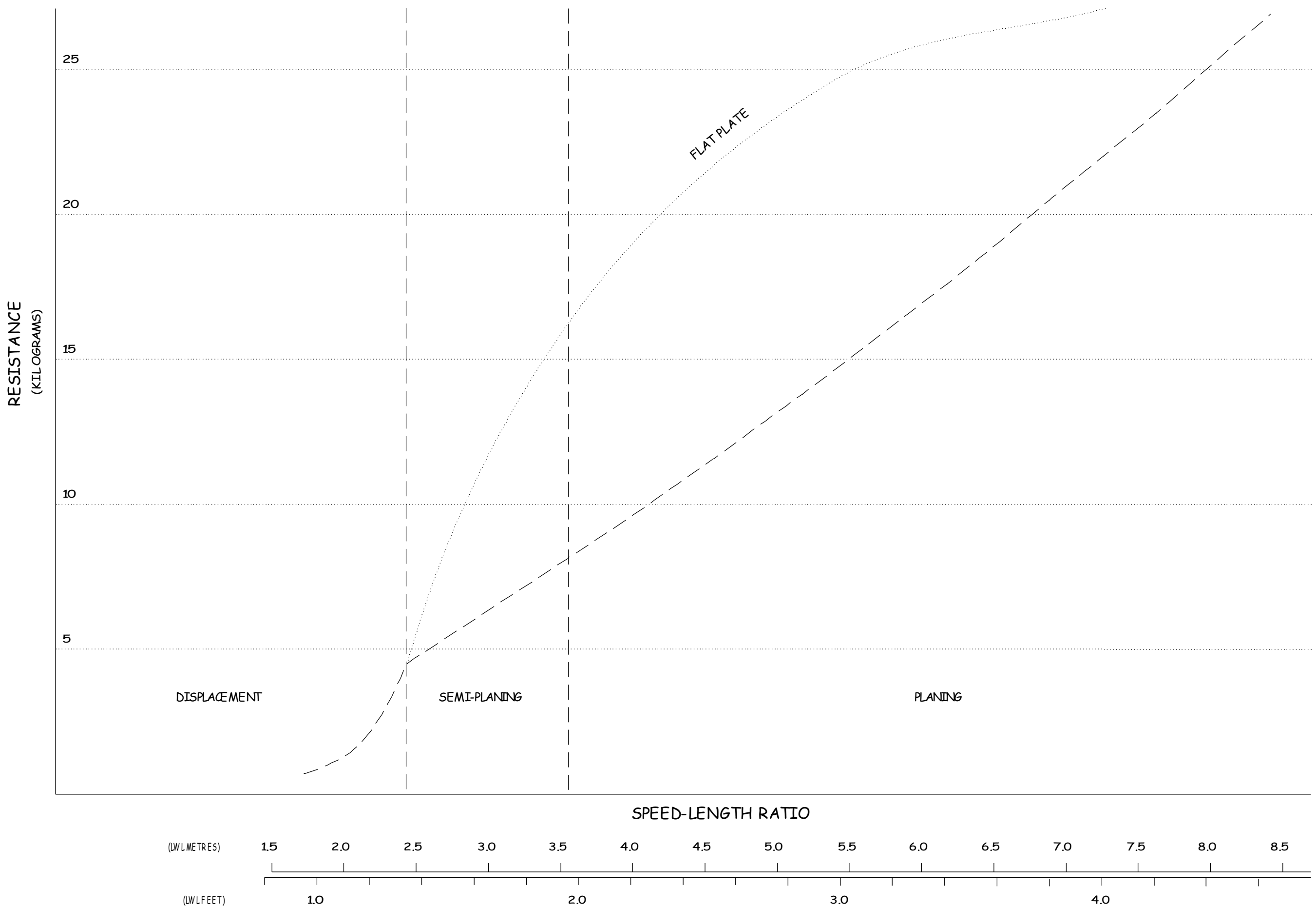


Figure A1-5

NS14 - 225S : A PRELIMINARY DESIGN FOR DWL 6580 USING CIRCULAR CROSS-SECTIONS

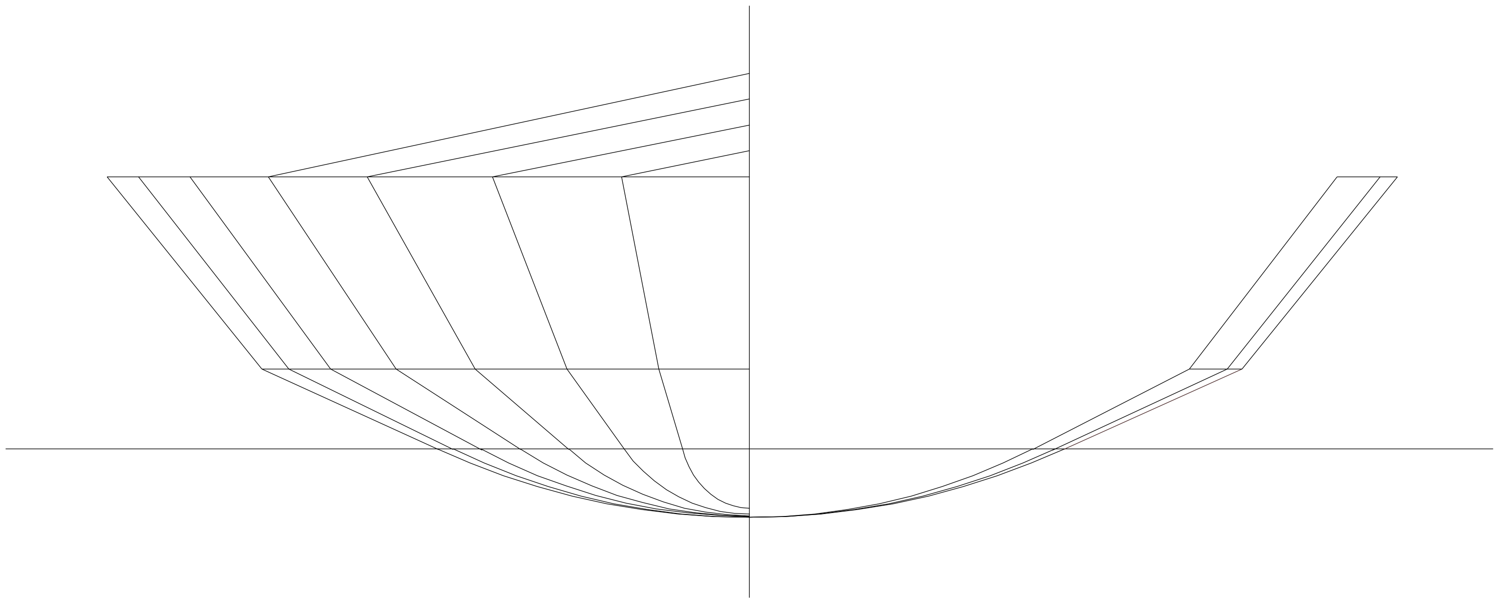


Figure A1-6